INTRODUCTION TO RADIO ASTRONOMY

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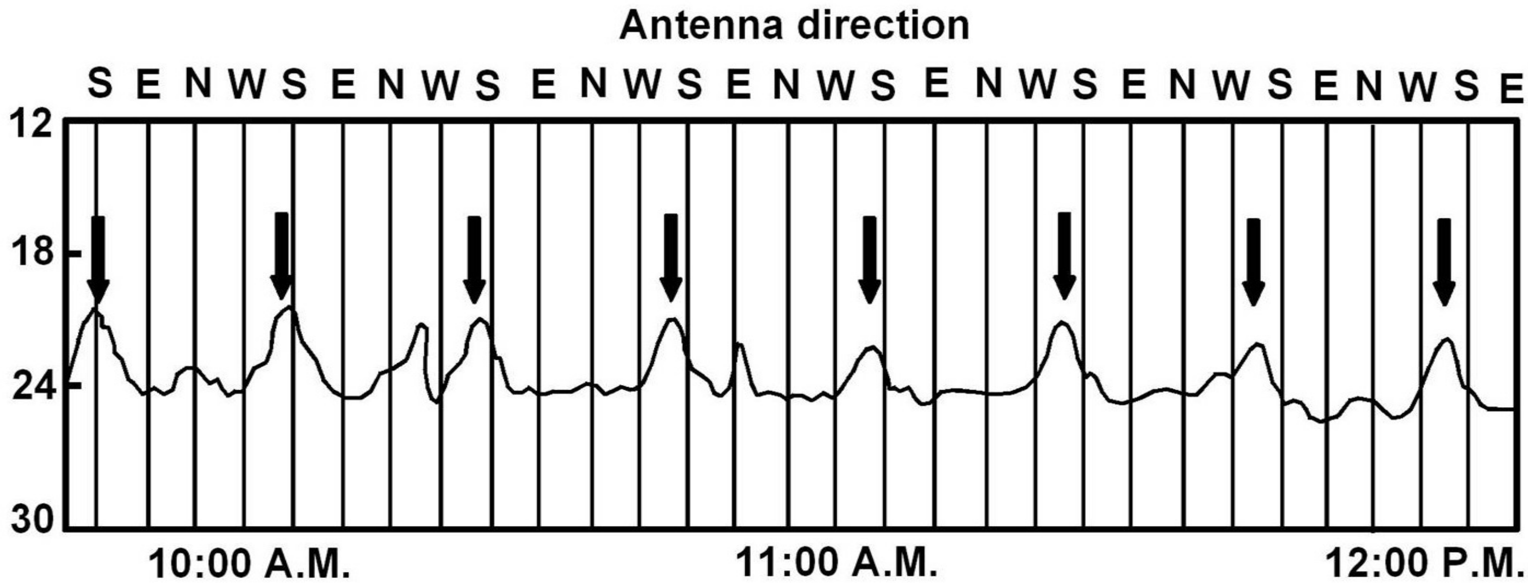
Sir Karl Jansky's records



He was trying to locate some sources of static noise that interfered with radio reception at 14.6 m and at 10 m wavelengths. Some of the noise he found to be originating from nearby thunderstorms, while others were from very distant sources. He also listened to a faint hiss which was not related to the thunderstorms at all. He recorded this data over a year and noted the crests and troughs in the intensity of the weak signal.... Changing direction of the peaks (~22 min. intervals) indicate that the source is not stationary w.r.t. Earth, i.e. having extraterrestrial origin.

Commonly used units in Radio Astronomy

1 Astronomical Unit (AU) = 150,000,000 km
(Average distance between Sun and Earth.)

1 Parsec (pc) = 3.26156378 light years (~206265 AU)

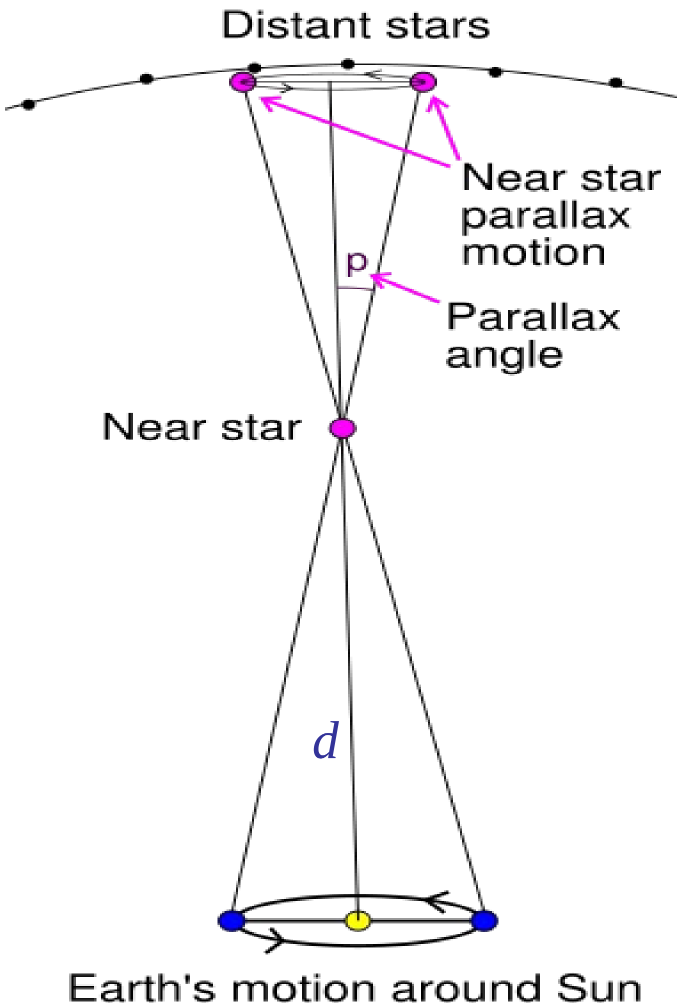
1 kpc = 1000 pc (kpc is used for measuring distances within our galaxy and neighboring galaxies)

1 Mpc = 1,000,000 pc (Mpc is used for measuring distances in cosmology)

1 Solar flux unit (SFU) = 10^{-22} W m^{-2} Hz^{-1} (used for measuring radio flux densities of the Sun)

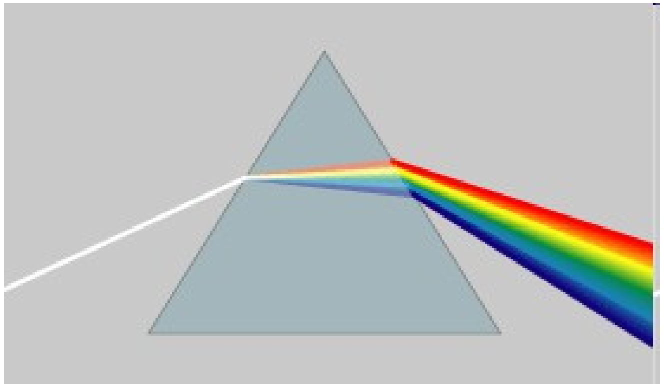
1 jansky = 10^{-26} W m^{-2} Hz^{-1} (used for measuring flux density in Radio Astronomy)

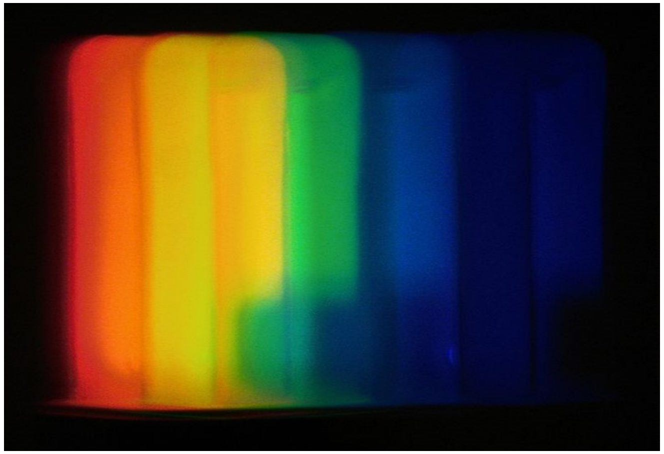
Explanation of some Units



d(pc) = 1 / p (arcsec)

(a) Explanation of unit 'pc'



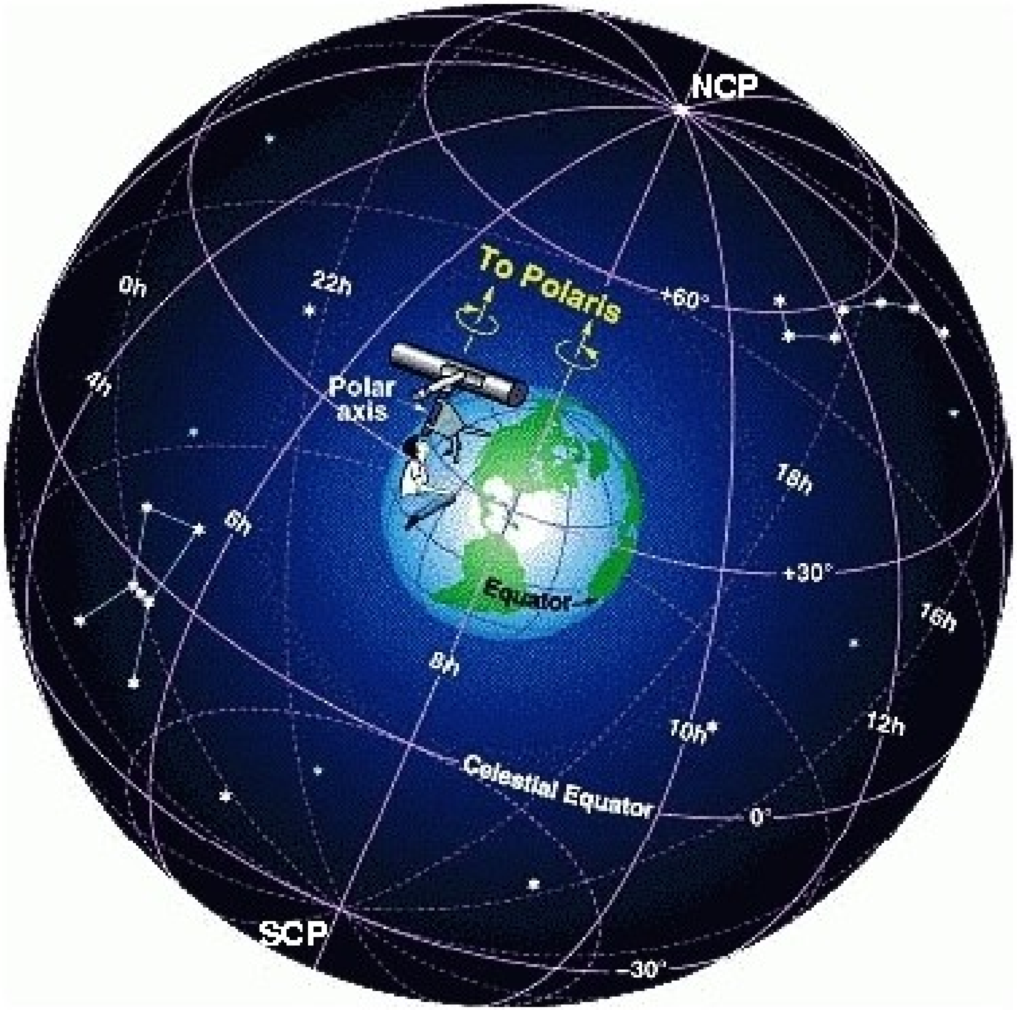


Intensity of light is distributed as a function of wavelengths

(a) Explanation of unit 'jansky'

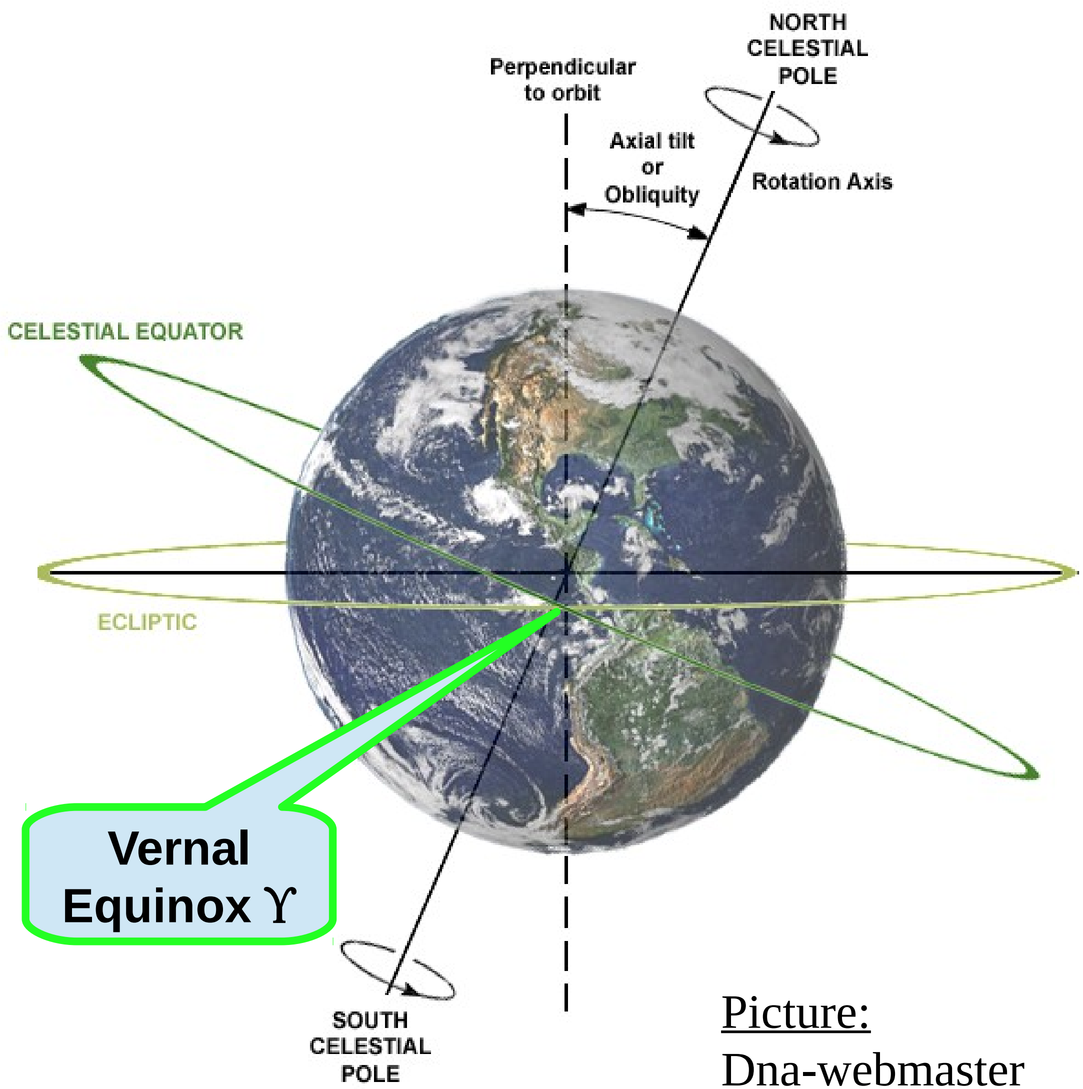
The Celestial Sphere

Consider a sphere of infinite radius with Earth at its center. All celestial objects like stars and galaxies appear to lie on the surface of this sphere known as ***celestial sphere***. Two angular coordinates are required to specify any point like a star. Intersection of a plane passing through center forms a ***great circle***. Planes not passing through center forms ***small circle***.



The Ecliptic and the Celestial Equator

Consider the Earth as the center of the Universe. From Earth, the Sun seems to revolve around it in a path called ***ecliptic***. The celestial sphere seems like a shell rotating around the Earth on the North-South axis. Extending the equator of Earth on the celestial sphere forms the ***celestial equator***.

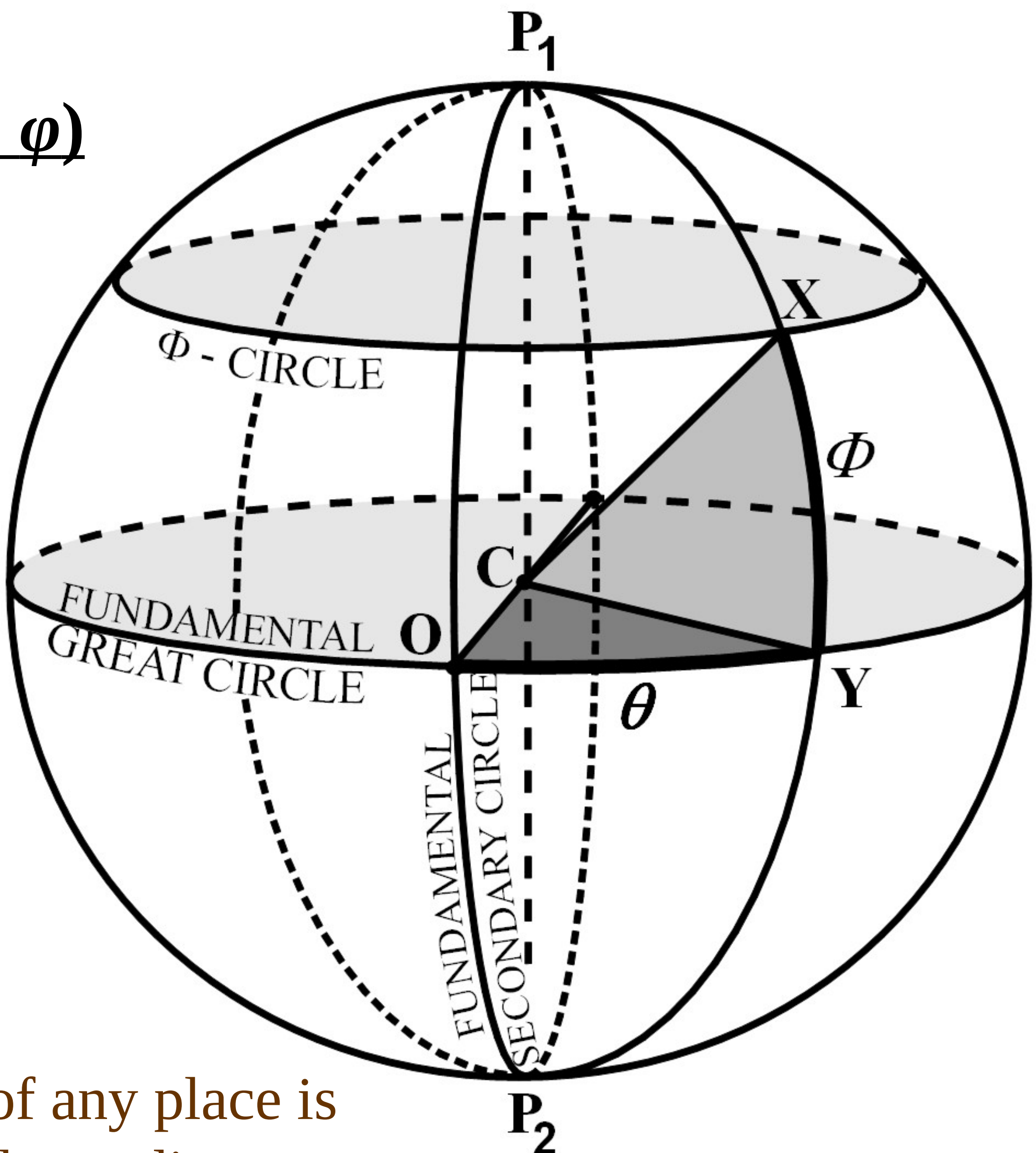


Picture:
Dna-webmaster

Coordinate Systems used in Radio Astronomy

A general Spherical Coordinate System (θ, φ)

Consider a point X on the celestial sphere. The coordinates of X are $\theta = OY$ (measured along the fundamental great circle) and $\varphi = YX$ (measured along the secondary great circle). The loci of constant φ are small circles parallel to fundamental great circle known as the ***φ-circles***.

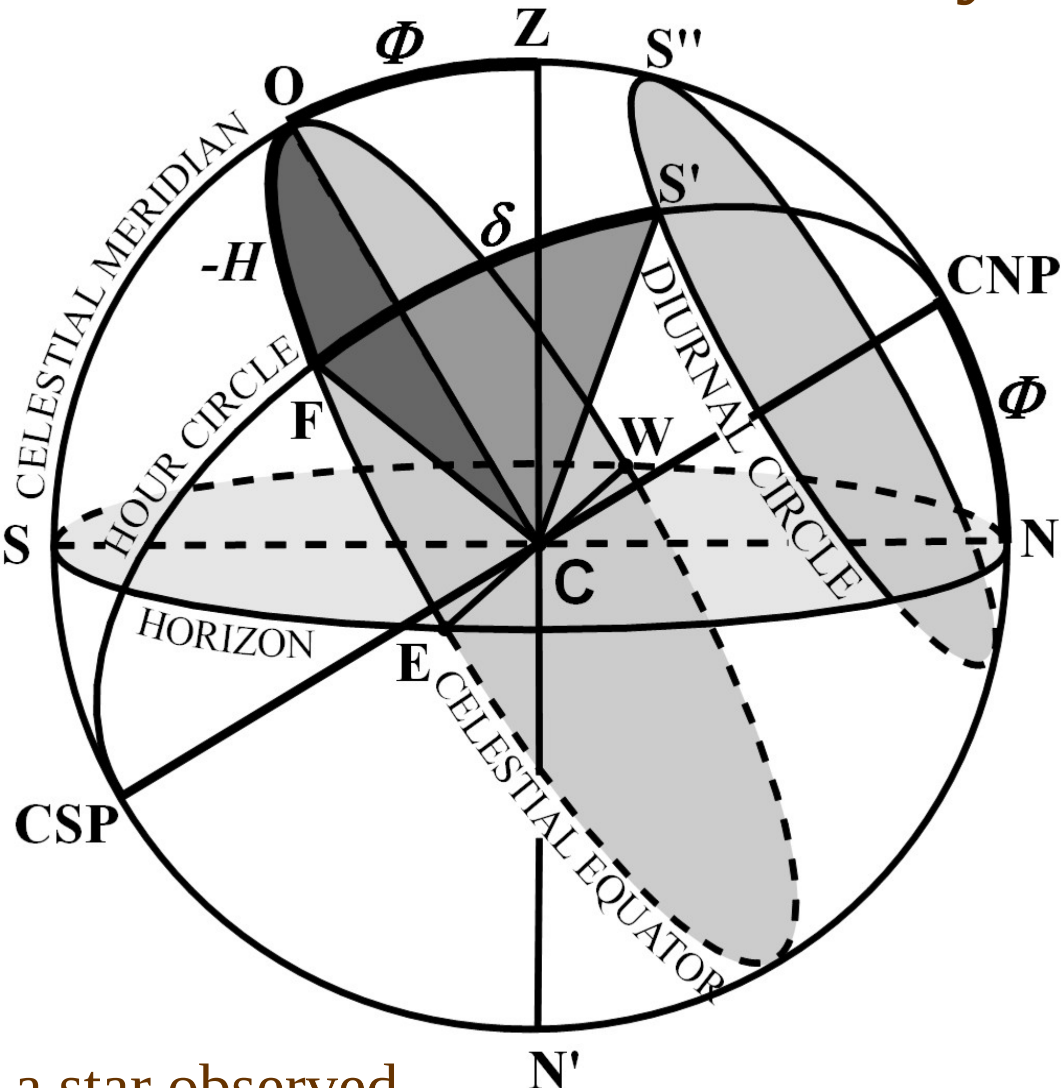


The latitude and longitude of any place is a good example of spherical coordinates.

Coordinate Systems used in Radio Astronomy

The Local Equatorial System (H, δ)

The plane of Earth's equator forms the ***celestial equator***. Small circles parallel to celestial equator form ***diurnal*** or ***declination circles***. Hour circle passing through the local zenith **Z,** nadir **N′** and celestial poles is the ***celestial meridian***.



If H_1 and H_2 are hour angles of a star observed simultaneously at two places having geographical longitudes l_1 and l_2 respectively we have $H_2 - H_1 = l_2 - l_1$

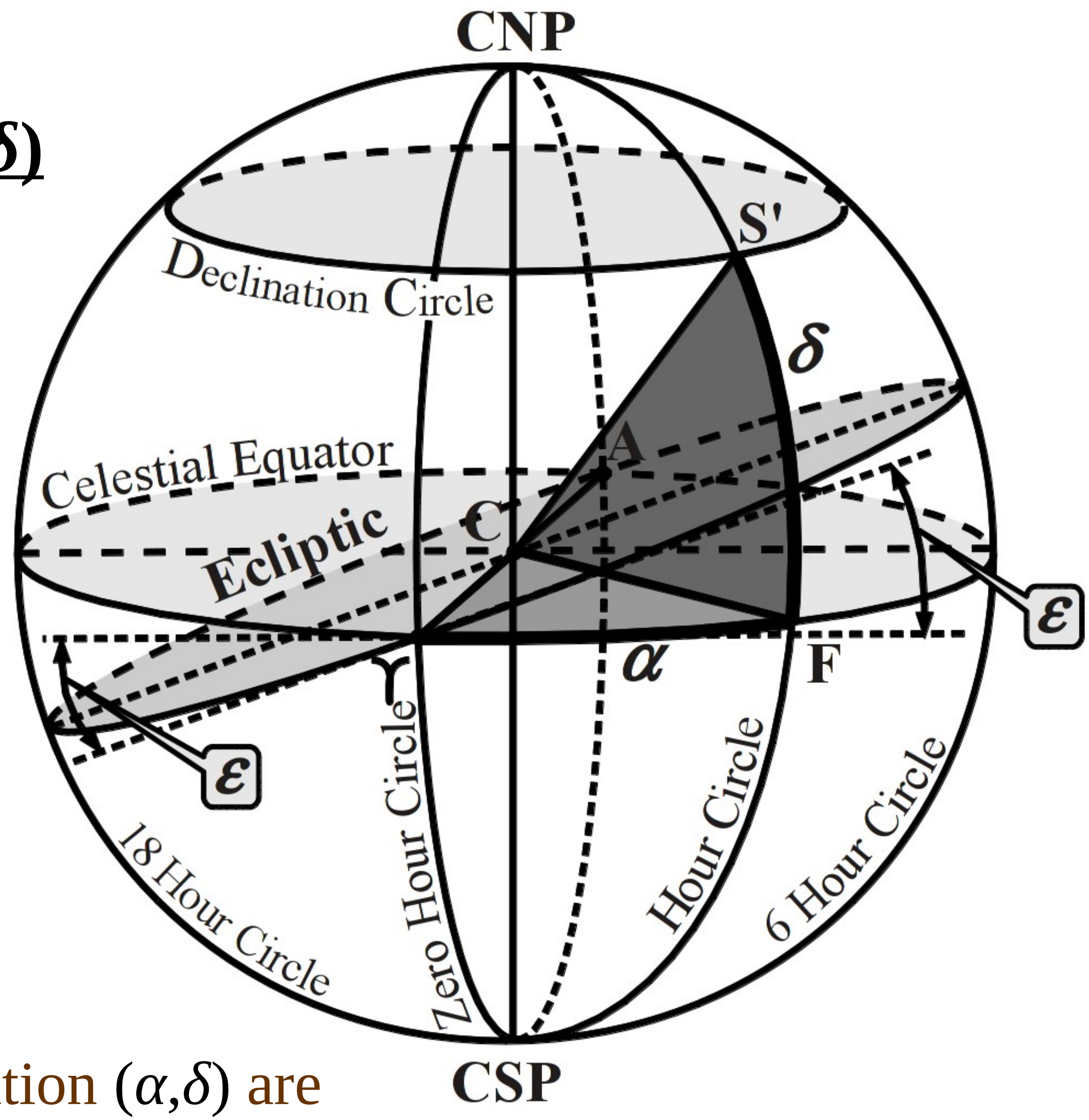
Coordinate Systems used in Radio Astronomy

The Universal Equatorial System (α, δ)

This is similar to the *local equatorial system* except for the reference point on celestial equator is fixed on the ***vernal equinox*** Υ (point at which ecliptic crosses the equator from South to North).

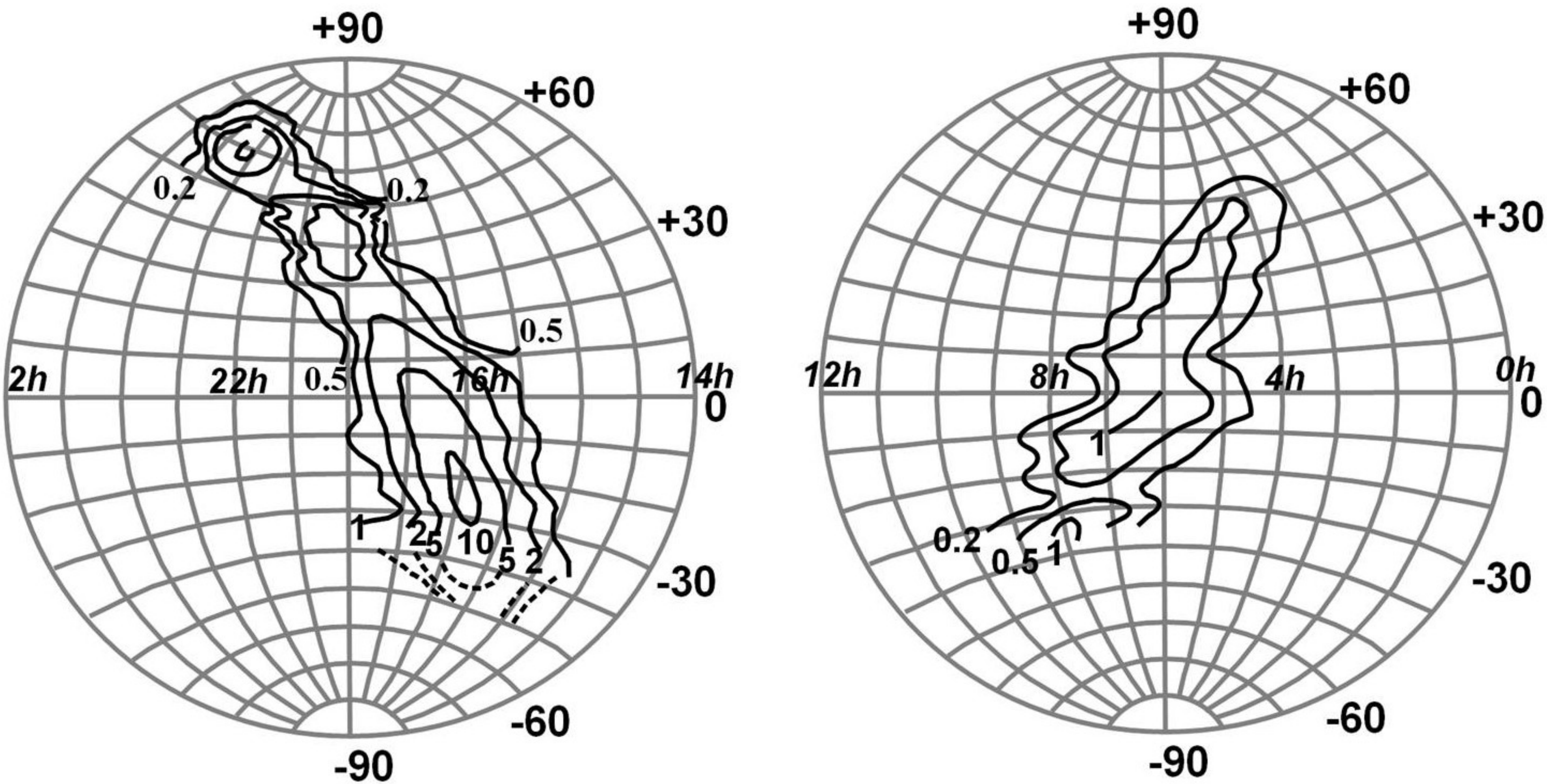
Obliquity of ecliptic

$$\varepsilon = 23.5^\circ$$



Right ascension and declination (α,δ) are used for preparing permanent star charts.

Reber's Radio map of Galaxy (1.87m)

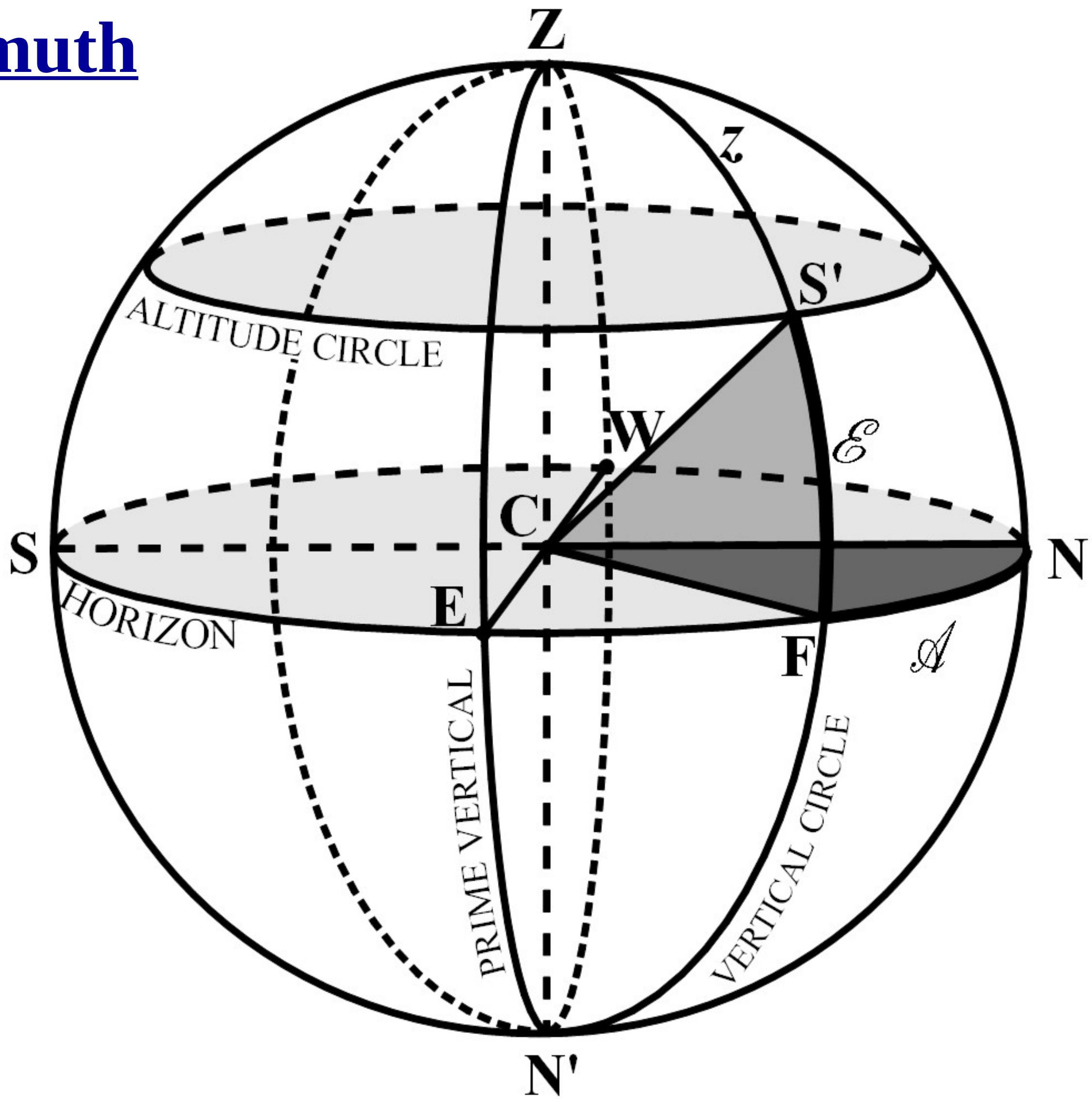


Reber's Radio map of Galaxy at 160 MHz (1.87m). Contours are in equatorial coordinates and are more or less coincident with the Milky Way (Adapted from Reber, 1944).

Coordinate Systems used in Radio Astronomy

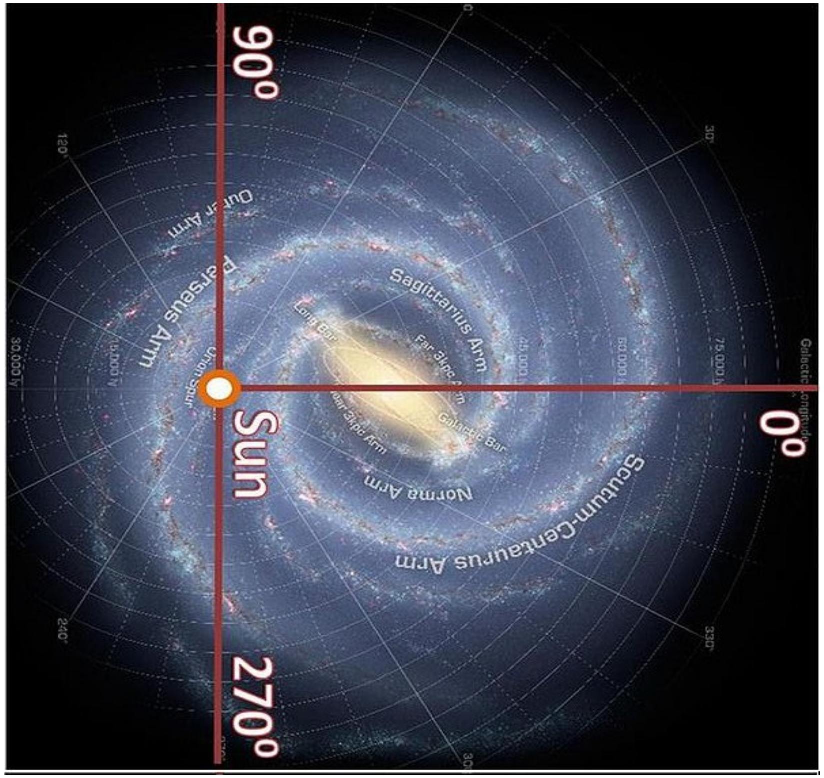
The Horizon or Altazimuth System (A ,E)

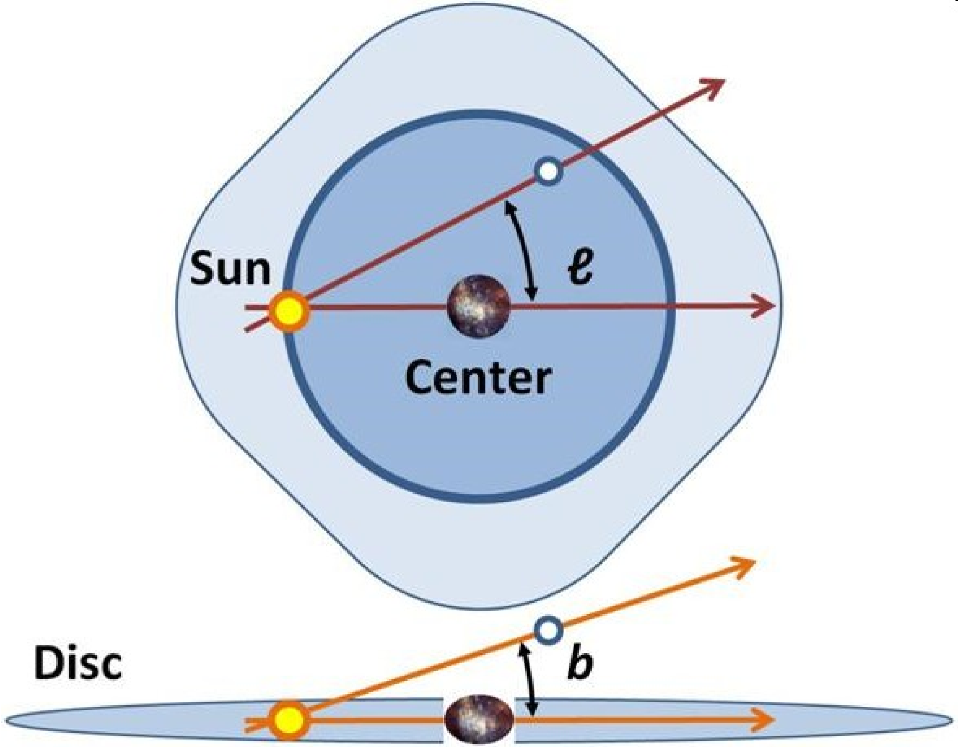
Any great circle touching the zenith **Z** and the nadir **N** is known as ***vertical circle***. A vertical circle joining the East and West (E and W) is known as the ***prime vertical***. Small circles parallel to the horizon are known as ***altitude circles***. Azimuth angle is **A** and altitude angle is **E.** The origin can be either N or S.



Useful for mechanical positioning a telescope antenna.

Foundation of Galactic Coordinates





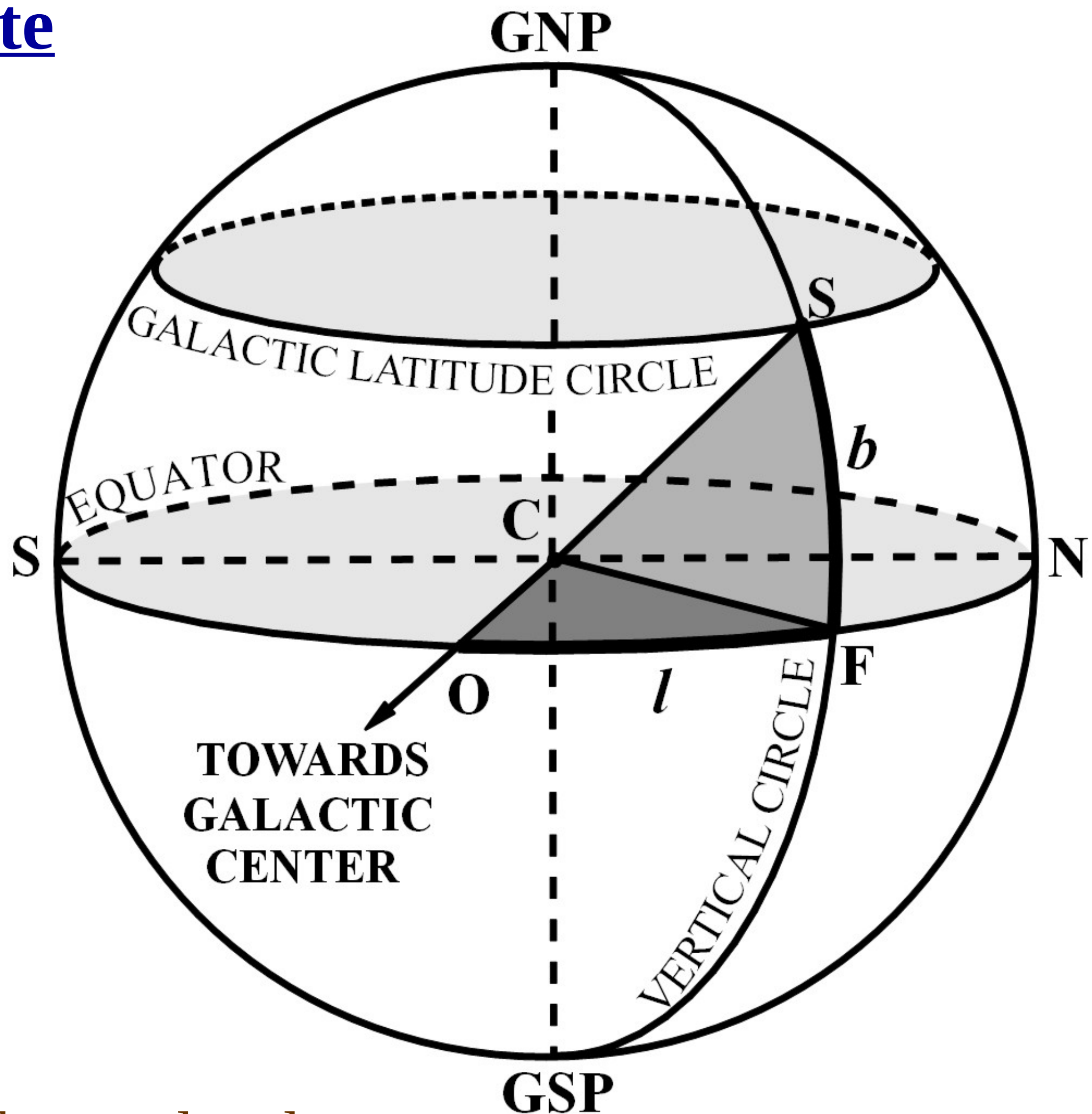
Artist's depiction of the Milky Way galaxy, showing the galactic longitude relative to the Sun.

Galactic coordinates use the Sun as vertex. Galactic longitude ℓ is measured with reference to the direction of center. Galactic latitude b is measured between the object and the galactic plane.

Coordinate Systems used in Radio Astronomy

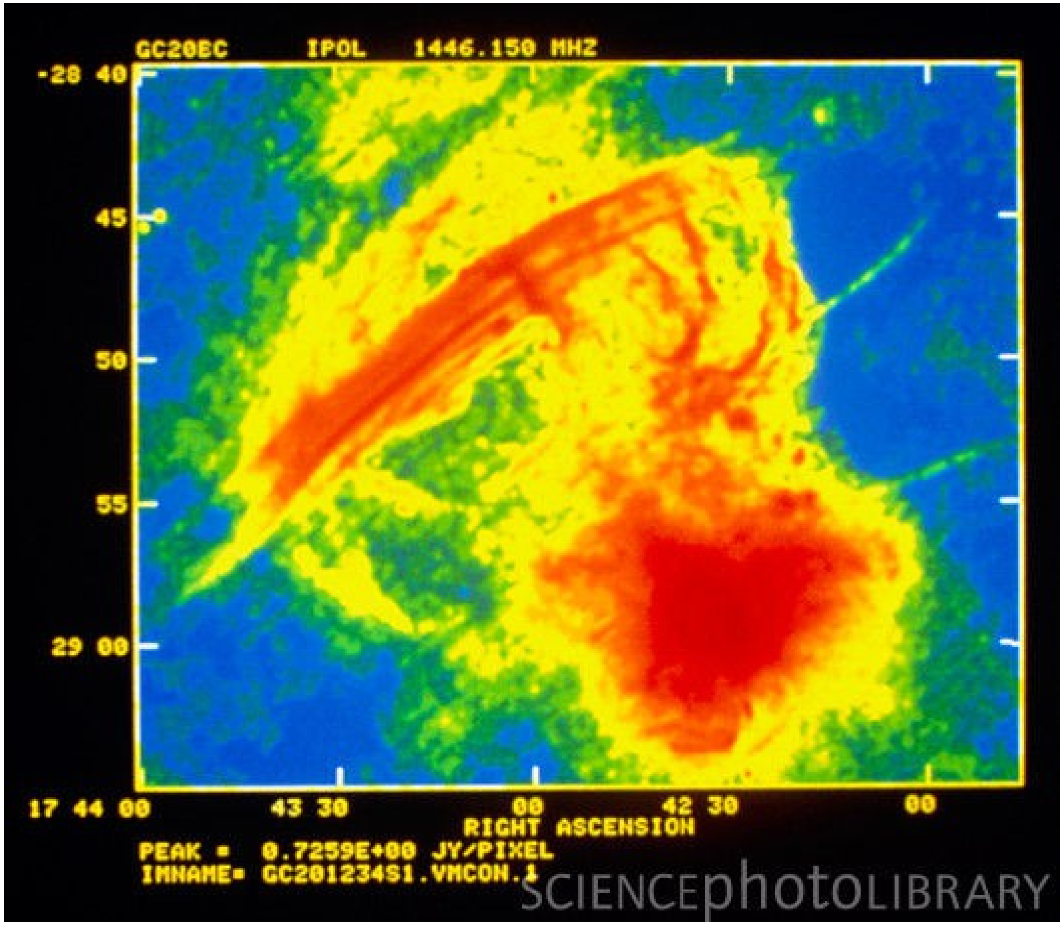
The Galactic Coordinate System (*l*, *b*)

The great circle formed by extending the plane of the Milky Way is the ***galactic equator***. The poles are ***Galactic North Pole*** GNP and ***Galactic South Pole*** GSP. The great half circles between the poles are galactic longitudes *l*. Small circles parallel to the galactic equator are galactic latitudes *b*.



Useful for studying the problems related to structure and motion of the Milky Way.

Some Radio Astronomical Sources



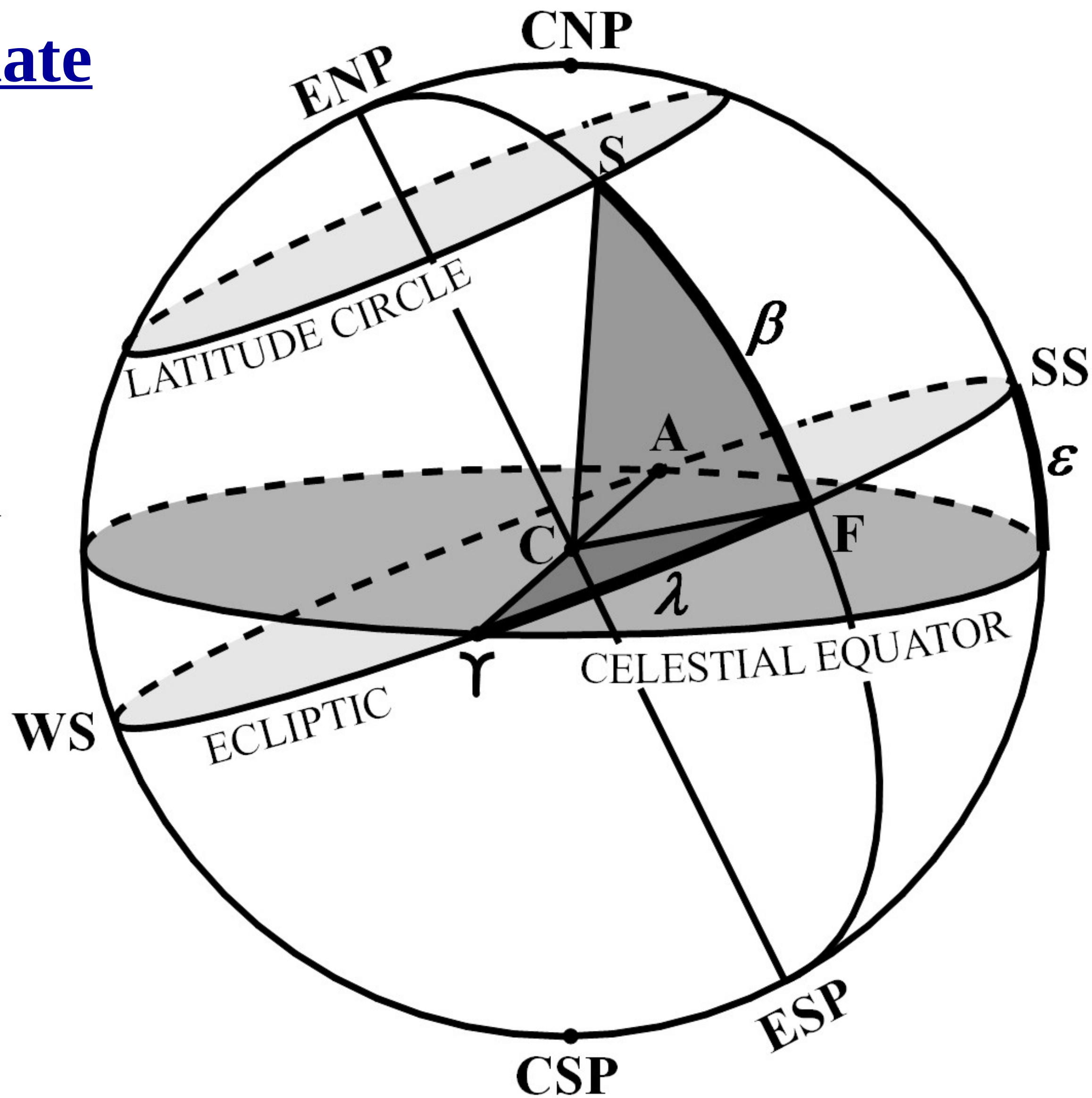
The **Galactic Center** is the rotational center of the Milky Way galaxy. Because of interstellar dust along the line of sight, the Galactic Center cannot be studied at visible, ultraviolet or soft X-ray wavelengths.

Radio image of the region surrounding the radio source Sagittarius A in the centre of our Galaxy. The colour code runs from red for the brightest regions through yellow, green and blue for the faintest parts. The area shown is about 180 light years across.

Coordinate Systems used in Radio Astronomy

The Ecliptic Coordinate System (λ, β)

The ecliptic forms the fundamental great circle with vernal equinox ϒ as its origin. The great half circles joining the ***Ecliptic North Pole*** ENP and the ***Ecliptic South Pole*** ESP are the celestial longitudes. Longitude of a star is λ (angle ϒCF) an its latitude is β (angle FCS).

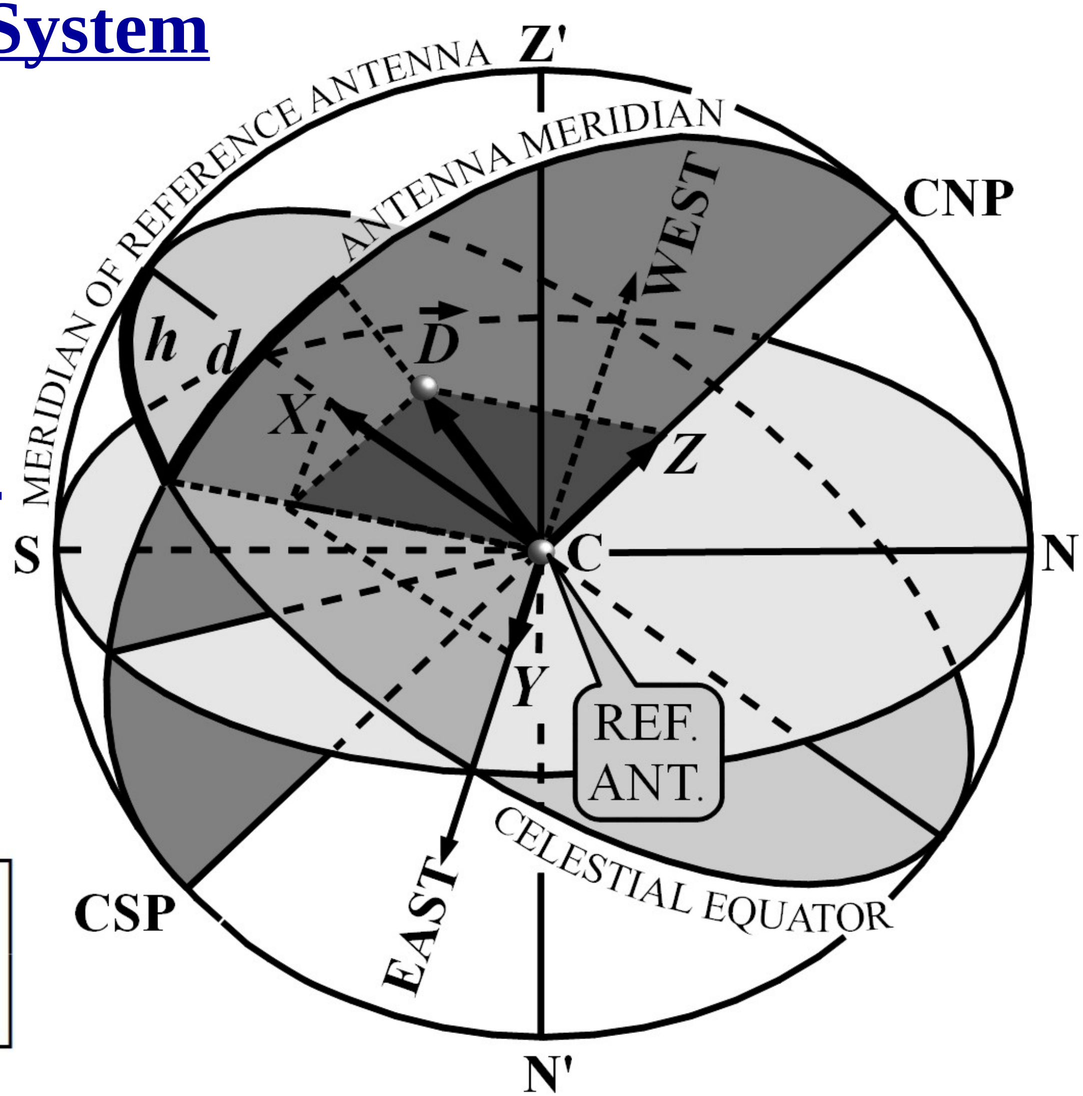


Useful for studying objects within the solar system.

Coordinate Systems used in Radio Astronomy

The *X*, *Y*, *Z* Coordinate System

A reference antenna is the origin C. Position of all other antennas are determined as *X*, *Y* and *Z*. ***D*** is position vector (***baseline vector***) from reference antenna to any other antenna making an hour angle *h* and declination *d*.



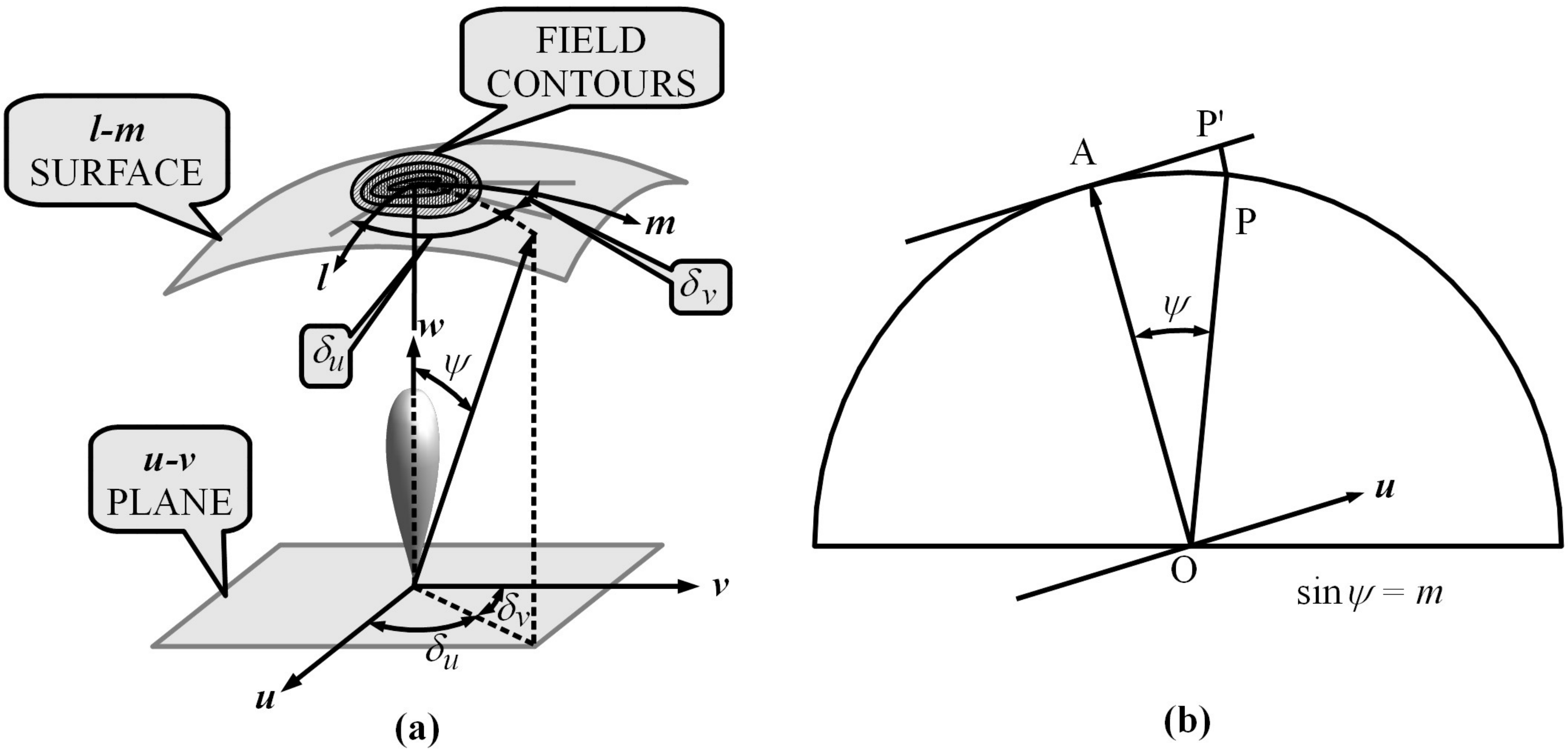
$$D = (X^2 + Y^2 + Z^2)^{0.5}$$

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = D \begin{bmatrix} \cos d \, \cos h \\ -\cos d \, \sin h \\ \sin d \end{bmatrix}$$

Used for determining relative the positions of antennas within an antenna array of a radio telescope.

Coordinate Systems used in Radio Astronomy

The *l*, *m* coordinate system and approximations



(a) Scanning a radio source located on the celestial sphere using a tracking antenna.

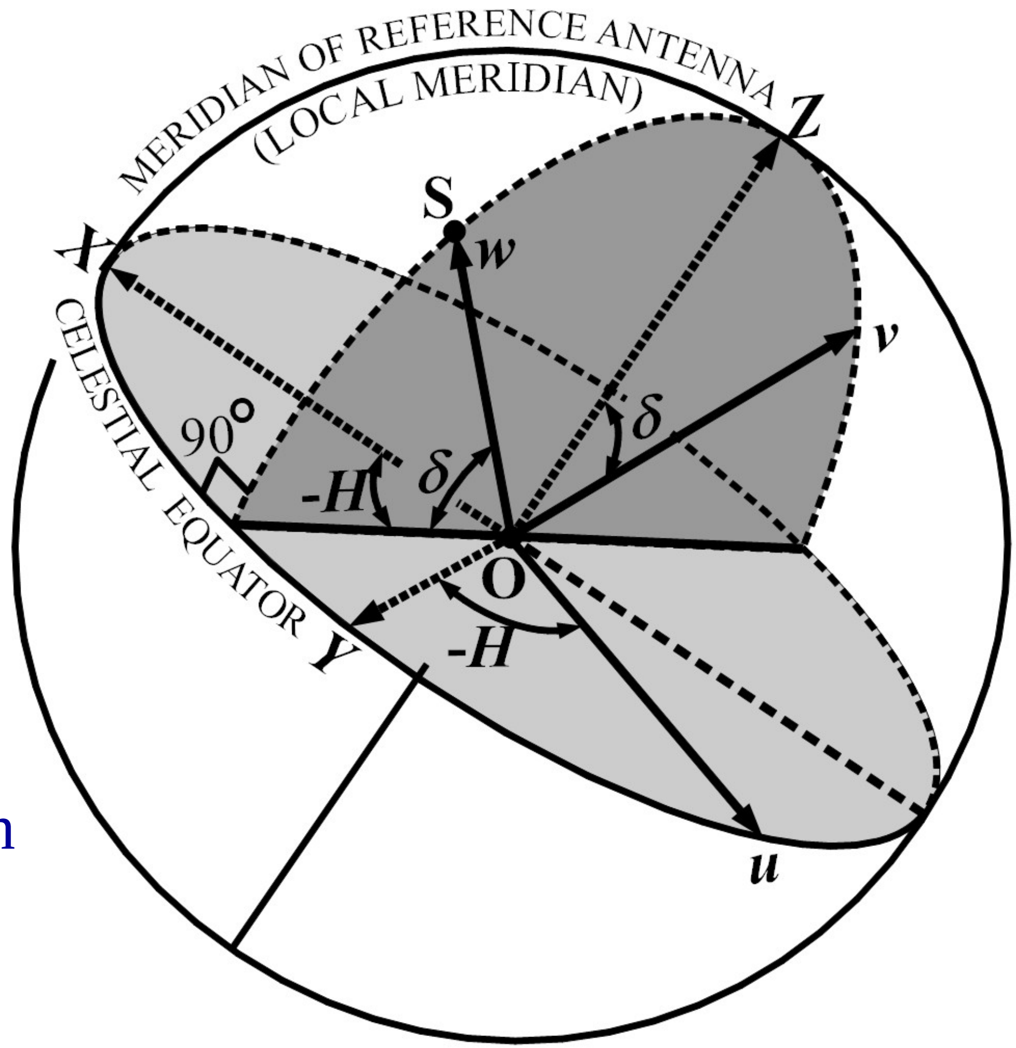
(b) Approximation of acutual field point P' to P.

$$m \approx \sin\psi \cos\delta_v \qquad l \approx \sin\psi \cos\delta_u$$

Coordinate Systems used in Radio Astronomy

The *u,v,w* Coordinate System and *X,Y,Z*

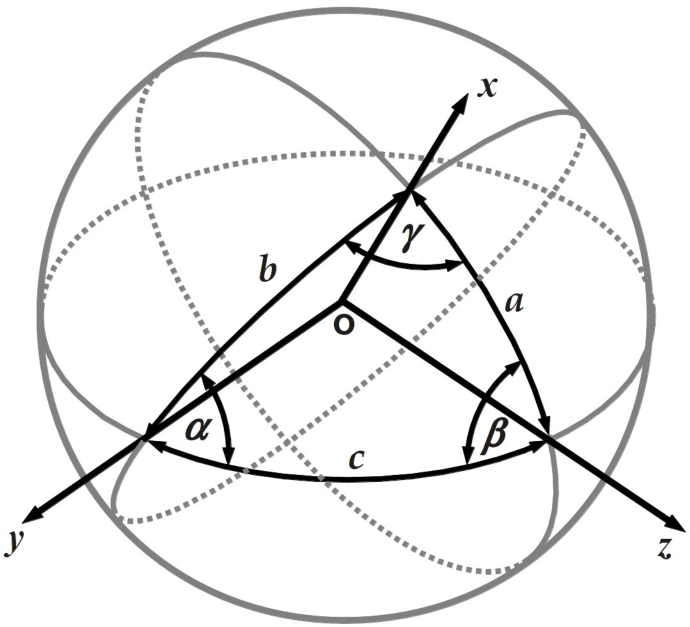
The *w* coordinate is always directed to the source **S**, and *u* always lies on the equatorial plane on the eastern side of **S**. The *u*−*v* plane is always perpendicular to the source direction *w*. *H* is local hour angle of the phase reference position on the radio source with respect to local meridian. Here, **S** is in eastern hemisphere and so *H* is negative.



The *u,v,w* coordinates are used by radio interferometers for super-synthesis in radio astronomy.

Basis of Coordinate Conversions

The following laws are based on geometries formed by great circles.



Spherical Law of Sines

$$\frac{\sin a}{\sin \alpha} = \frac{\sin b}{\sin \beta} = \frac{\sin c}{\sin \gamma}$$

Spherical Law of Cosines

$$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha$$

Conversion between Coordinate Systems

1. *Conversion between Local and Universal Equatorial Systems*

The declination δ coordinate is a identical among both the systems.

The relation between hour angle H and right ascension α is given as

$$H = \text{Sidereal Time} - \alpha$$

Conversion between Coordinate Systems

2. Conversion between Local Equatorial and Altazimuth Systems

Conversion of local equatorial coordinates (δ, H) to altazimuth coordinates (E, A)

$$\cos\mathsf{E}\ \sin\mathsf{A} = -\cos\delta\,\sin H$$
$$\cos\mathsf{E}\ \cos\mathsf{A} = \sin\delta\,\cos\mathsf{L} - \cos\delta\,\sin\mathsf{L}\ \cos H$$
$$\sin\mathsf{E} = \sin\delta\,\sin\mathsf{L} + \cos\delta\,\cos\mathsf{L}\ \cos H$$

where, L is the latitude of the place.

Conversion of altazimuth coordinates (E, A) to local equatorial coordinates (δ, H)

$$\cos\delta\,\sin H = -\cos\mathsf{E}\,\sin\mathsf{A}$$
$$\cos\delta\,\cos H = \sin\mathsf{E}\,\cos\mathsf{L} - \cos\mathsf{E}\ \sin\mathsf{L}\ \cos\mathsf{A}$$
$$\sin\delta = \sin\mathsf{E}\,\sin\mathsf{L} + \cos\mathsf{E}\,\cos\mathsf{L}\ \cos\mathsf{A}$$

Conversion between Coordinate Systems

3. *Conversion between Universal Equatorial and Galactic Coordinates*

Let (α_g, δ_g) be the equatorial coordinates of the galactic pole and (α_c, δ_c) be those of the galactic center. We may relate the universal equatorial coordinates (α, δ) and galactic coordinates (l, b) using

$$\cos b \cos(l - l_\Omega) = \cos\delta \cos(\alpha - \alpha_g)$$

$$\cos b \sin(l - l_\Omega) = \sin\delta \cos\delta_g - \cos\delta \sin\delta_g \cos(\alpha - \alpha_g)$$

$$\sin b = \sin\delta \sin\delta_g + \cos\delta \cos\delta_g \cos(\alpha - \alpha_g)$$

where, l_Ω can be obtained from $\sin l_\Omega = -\sin\delta_c \sec\delta_g$

Conversion between Coordinate Systems

4. *Conversion between Universal Equatorial and Ecliptic Coordinates*

For conversion of universal equatorial coordinates (α, δ) to ecliptic coordinates (λ, β) one may use

$$\cos\beta \cos\lambda = \cos\delta \cos\alpha$$
$$\cos\beta \sin\lambda = \sin\delta \sin\varepsilon + \cos\delta \cos\varepsilon \sin\alpha$$
$$\sin\beta = \sin\delta \cos\varepsilon - \cos\delta \sin\varepsilon \sin\alpha$$

where, ε is the obliquity of the ecliptic.

For the reverse conversion one may use

$$\cos\delta \cos\alpha = \cos\beta \cos\lambda$$
$$\cos\delta \sin\alpha = -\sin\beta \sin\varepsilon + \cos\beta \cos\varepsilon \sin\lambda$$
$$\sin\delta = \sin\beta \cos\varepsilon + \cos\beta \sin\varepsilon \sin\lambda$$

Conversion between Coordinate Systems

5. *Conversion relations of X,Y,Z and u,v,w Coordinates*

The transformation of X, Y, Z to u, v, w is expressed as

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \sin H & \cos H & 0 \\ -\sin\delta\ \cos H & \sin\delta\ \sin H & \cos\delta \\ \cos\delta\ \cos H & -\cos\delta\ \sin H & \sin\delta \end{bmatrix} \begin{bmatrix} X_\lambda \\ Y_\lambda \\ Z_\lambda \end{bmatrix}$$

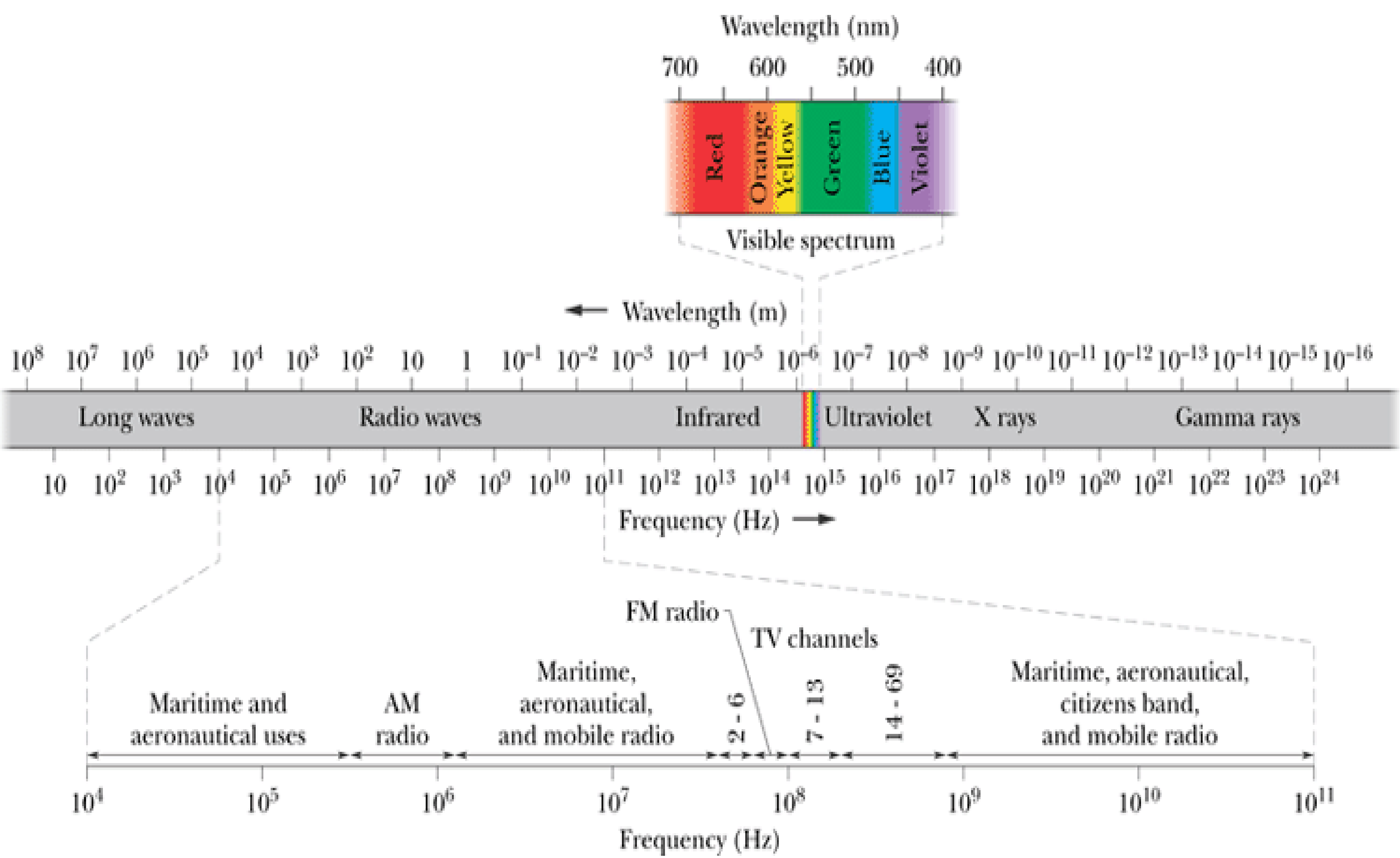
where, $X_\lambda = X/\lambda$, $Y_\lambda = Y/\lambda$, and $Z_\lambda = Z/\lambda$. The u, v, w coodinates can also be obtained from baseline length D_λ where $D_\lambda = D/\lambda$, λ is the wavelength and $D = (X^2 + Y^2 + Z^2)^{0.5}$.

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = D_\lambda \begin{bmatrix} \cos d\ \sin(H-h) \\ \sin d\ \cos\delta\ -\cos d\ \sin\delta\ \cos(H-h) \\ \sin d\ \sin\delta + \cos d\ \cos\delta\ \cos(H-h) \end{bmatrix}$$

The conversion to X, Y, Z from altazimuth system can done as

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = D \begin{bmatrix} \cos\mathcal{L}\ \sin\mathcal{E} - \sin\mathcal{L}\ \cos\mathcal{E}\ \cos\mathcal{A} \\ \cos\mathcal{E}\ \sin\mathcal{A} \\ \sin\mathcal{L}\ \sin\mathcal{E} + \cos\mathcal{L}\ \cos\mathcal{E}\ \cos\mathcal{A} \end{bmatrix}$$

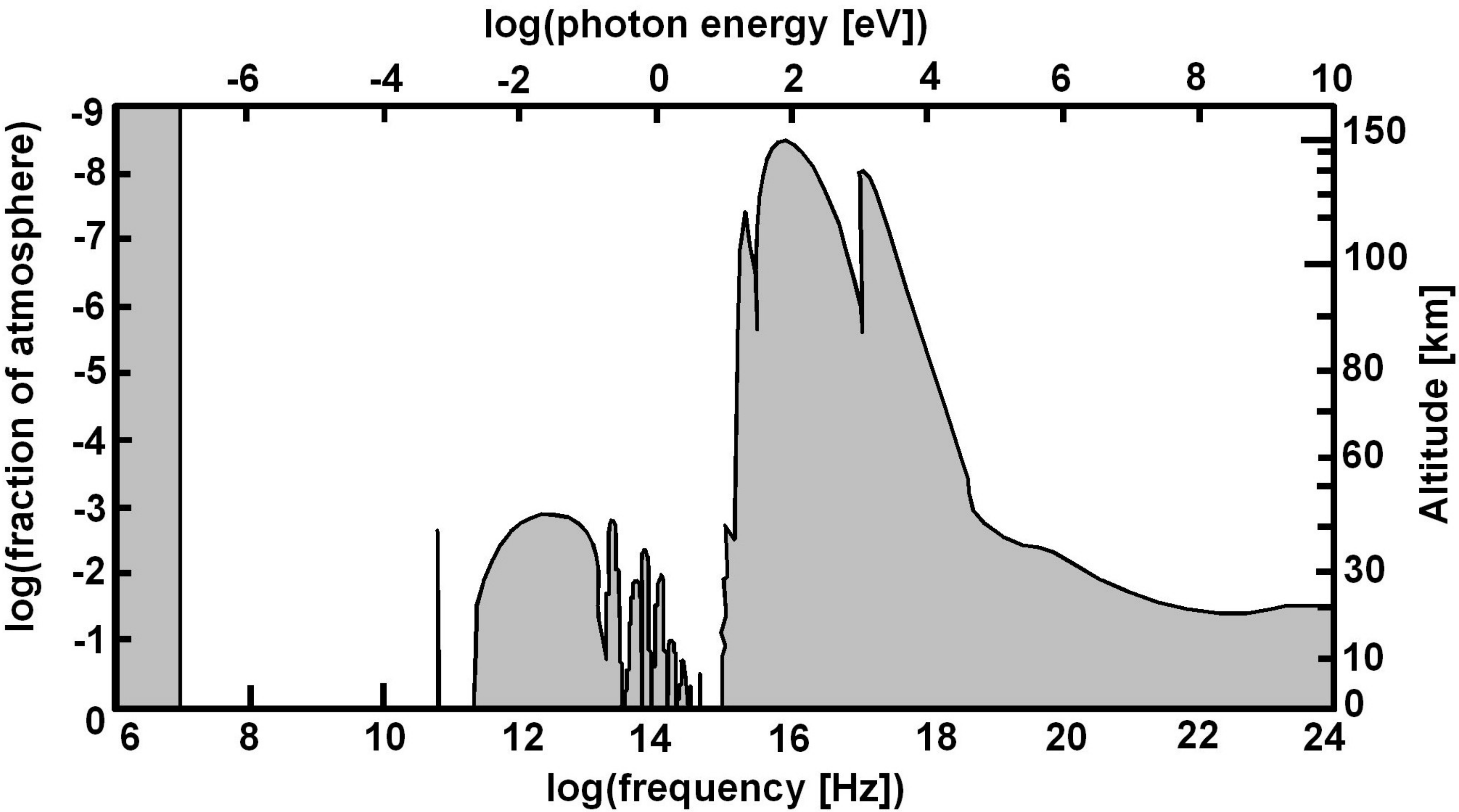
The Electromagnetic spectrum



Communication Radio bands

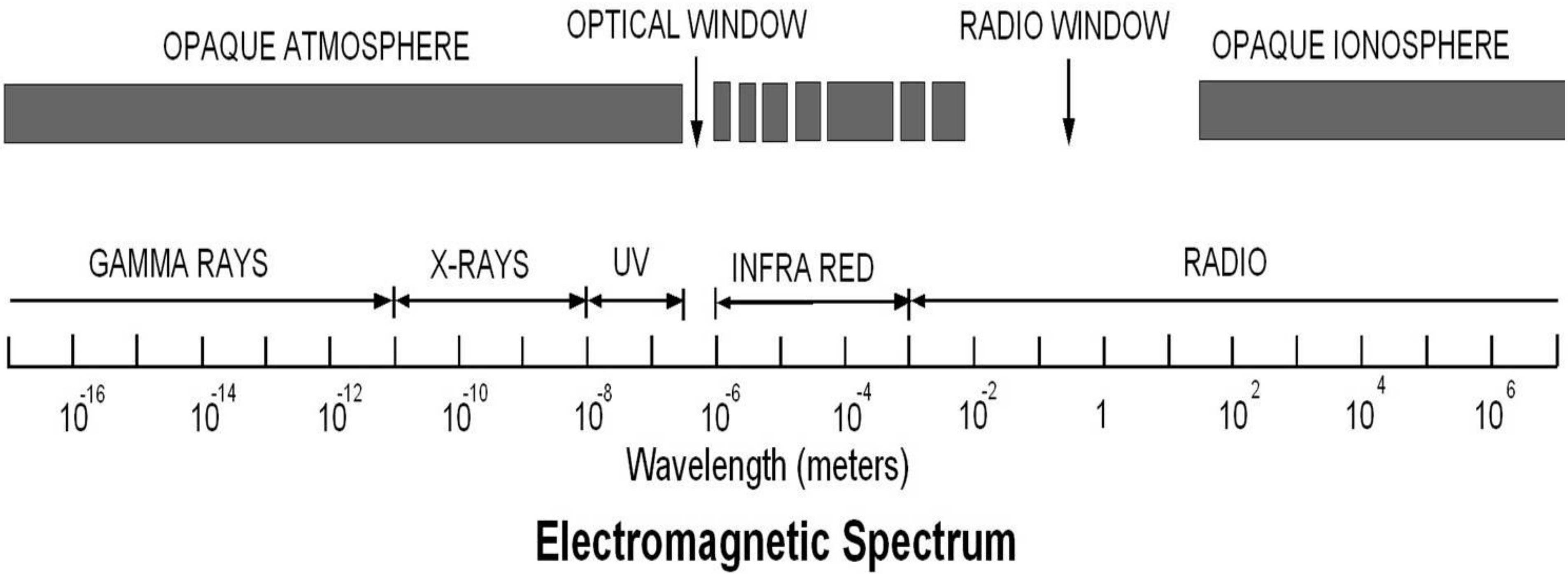
Band	Frequency	Wavelength
MF (medium freq.)	300 – 3000 kHz	1000 – 100 m
HF (high freq.)	3 – 30 MHz	100 – 10 m
VHF (very high freq.)	30 – 300 MHz	10 – 1 m
UHF (ultra high freq.)	300 – 3000 MHz	100 – 10 cm
SHF (super high freq.)	3 – 30 GHz	10 – 1 cm
EHF (extremely high freq.)	30 – 300 GHz	10 – 1 mm

Atmospheric Transparency of EM waves



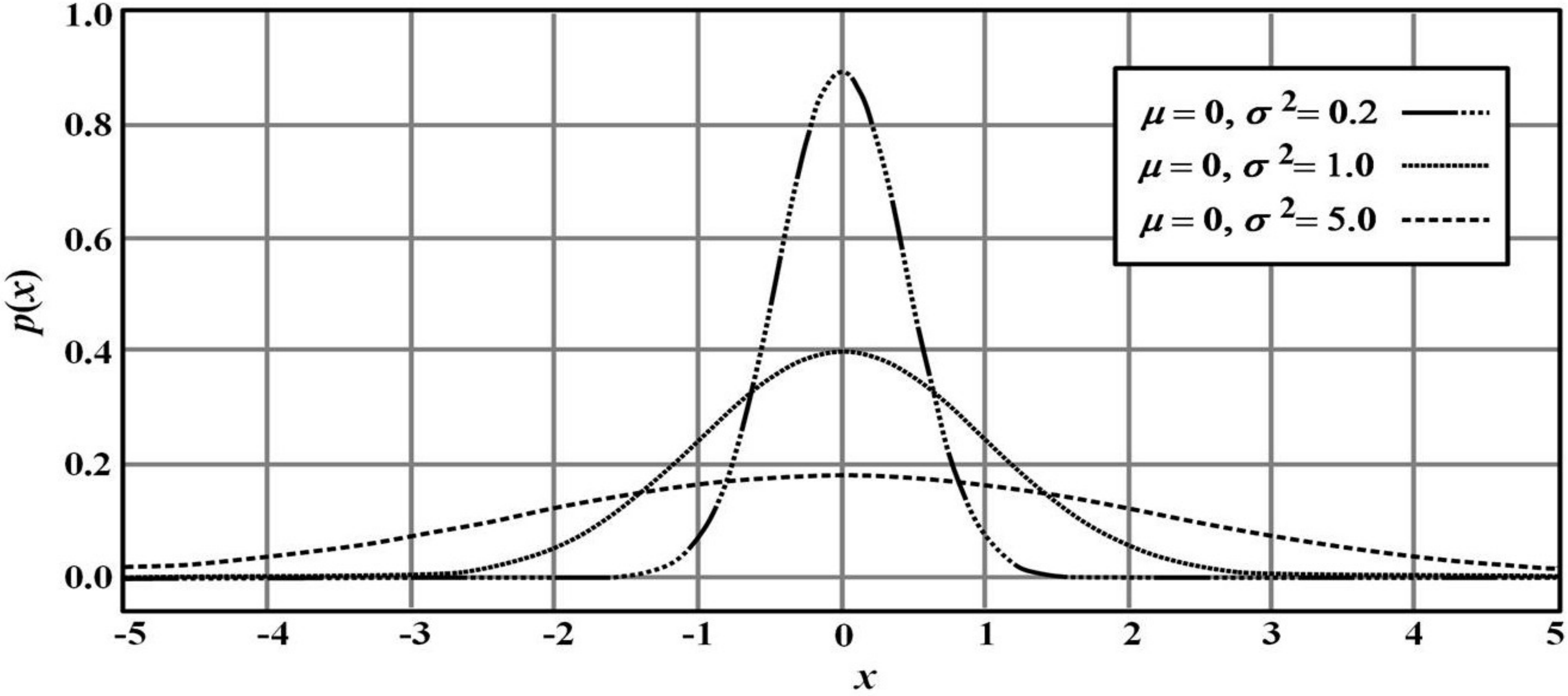
Atmosphere is transparent above the shaded area (Adapted from Giacconi et al. 1968).

Availability of EM Spectrum



Availability of Extra-Terrestrial Electromagnetic Spectrum on Earth.

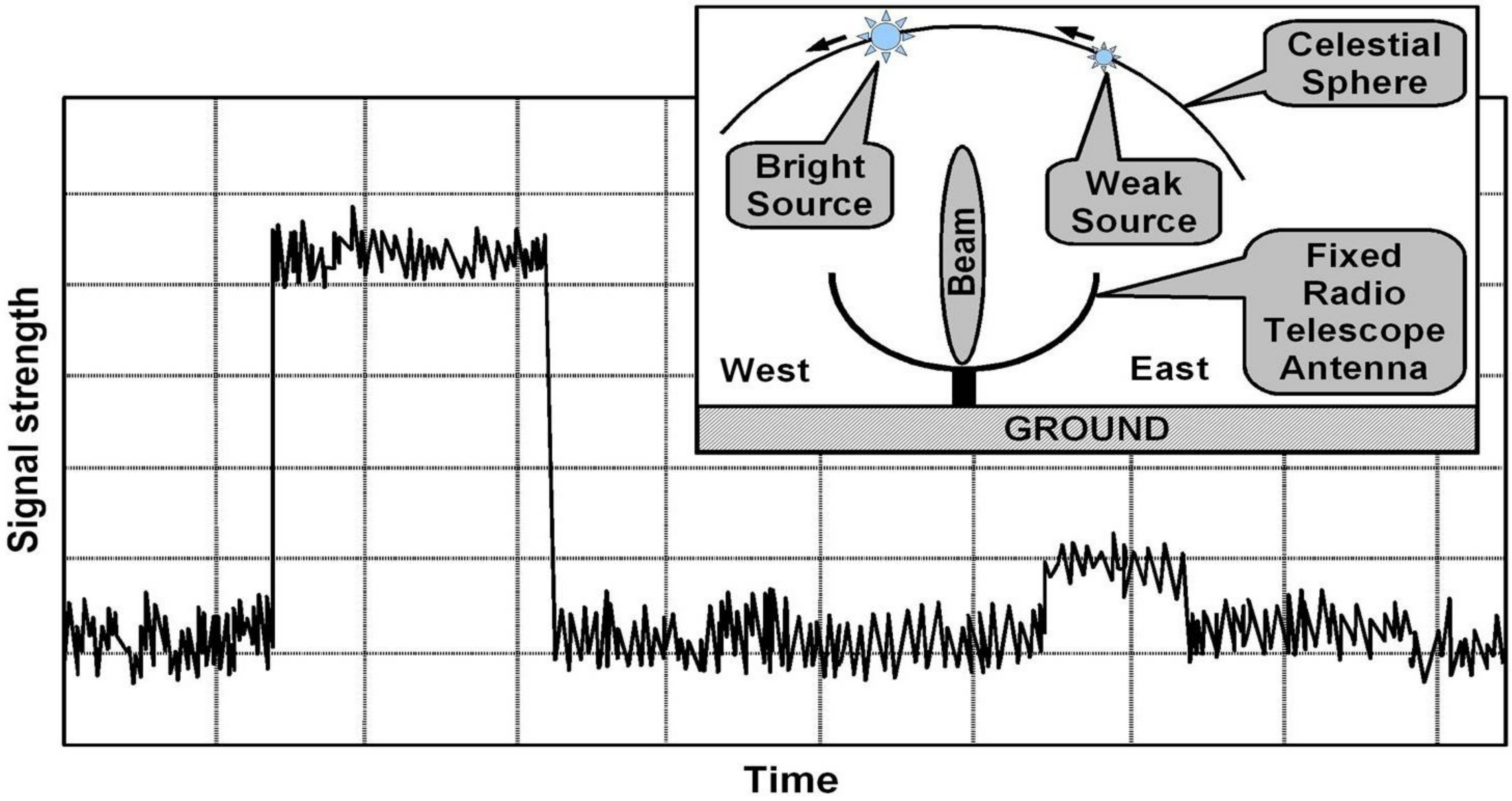
Nature of Radio Astronomical Signals



Most of the radio astronomical signals have random fluctuation of amplitude and phase. They have a Gaussian distribution. They are randomly polarized. Gaussian probability distribution for zero mean and 3 different variances (σ^2) are shown.

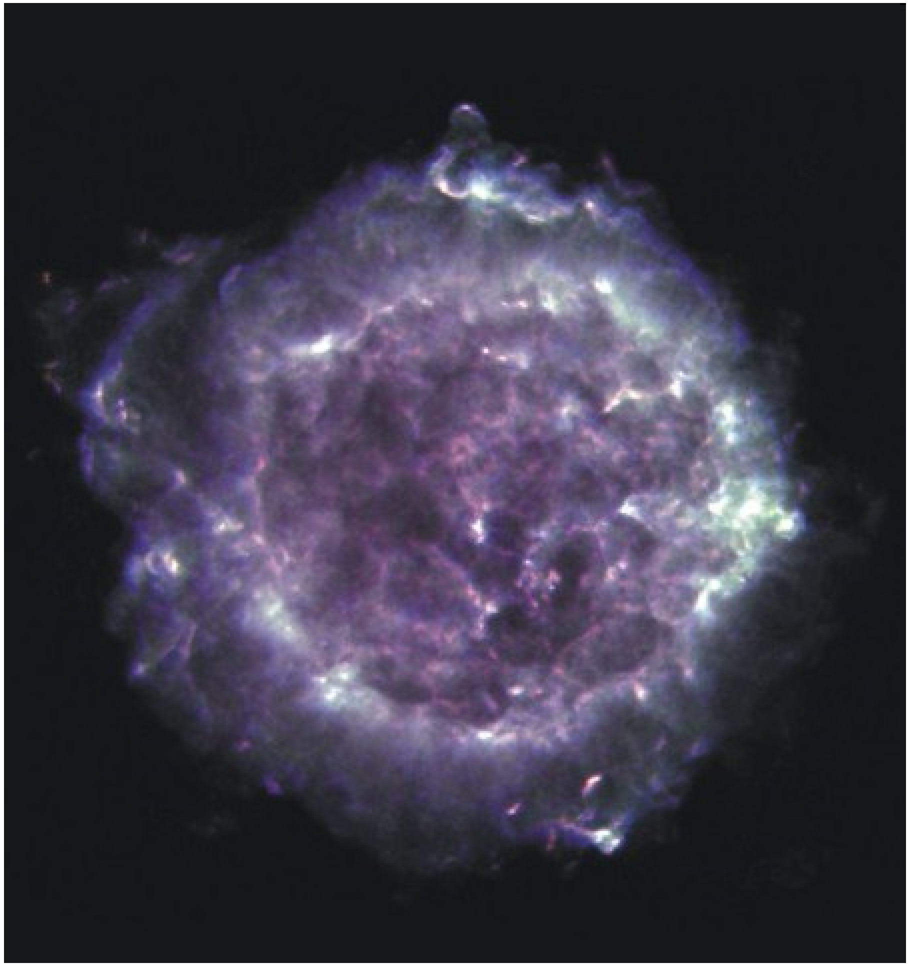
$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2} \right)$$

Drift curve from two radio sources



Two radio sources (strong and weak) producing a drift curve when they cross the antenna beam.

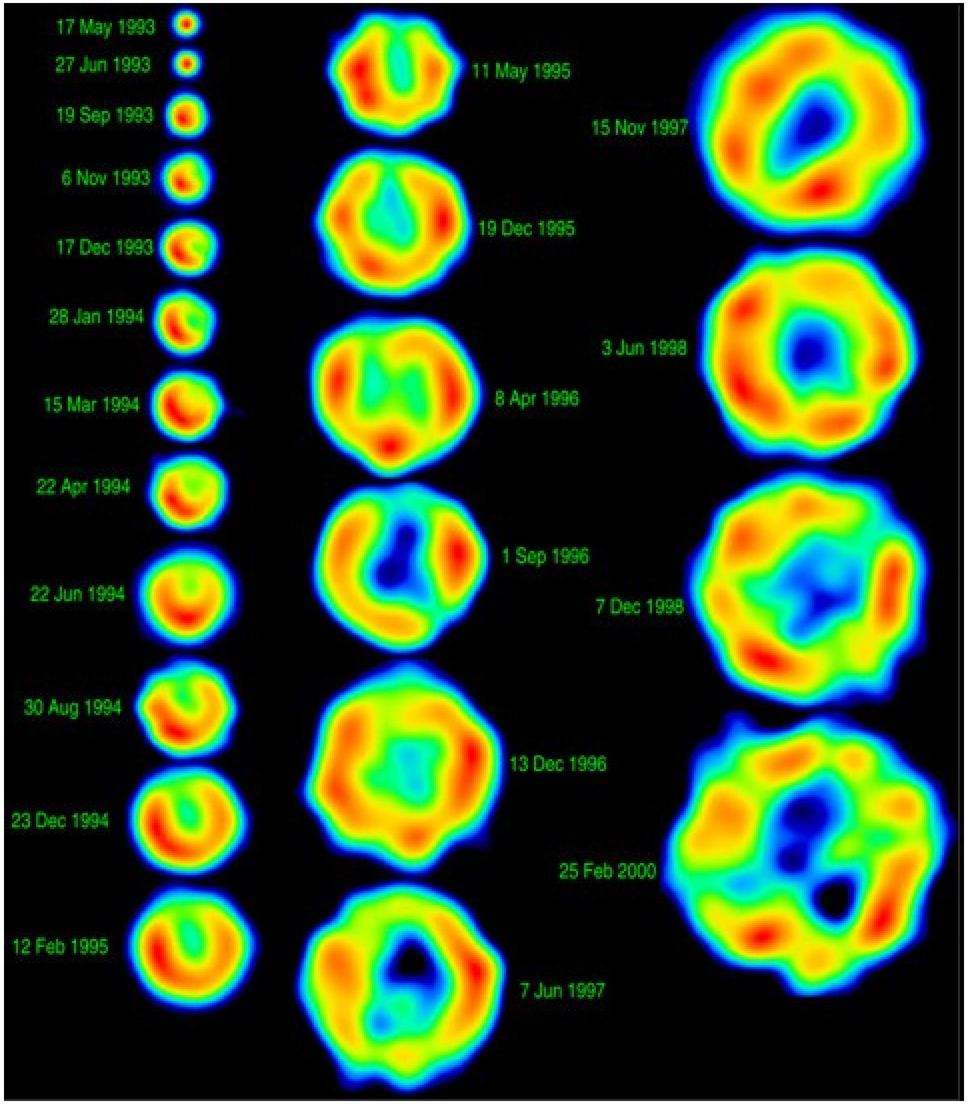
Some Radio Astronomical Sources



Cassiopeia A is the remnant of a supernova explosion that occured over 300 years ago in our Galaxy, at a distance of about 11,000 light years from us. Cassiopeia A is one of the brightest radio sources in the sky, and has been a popular target of study for radio astronomers for decades. The material that was ejected from the supernova explosion can be seen in this image as bright filaments.

Radio image using frequencies 1.4 GHz, 5.0 GHz and 8.4 GHz. Courtesy NRAO/AUI.

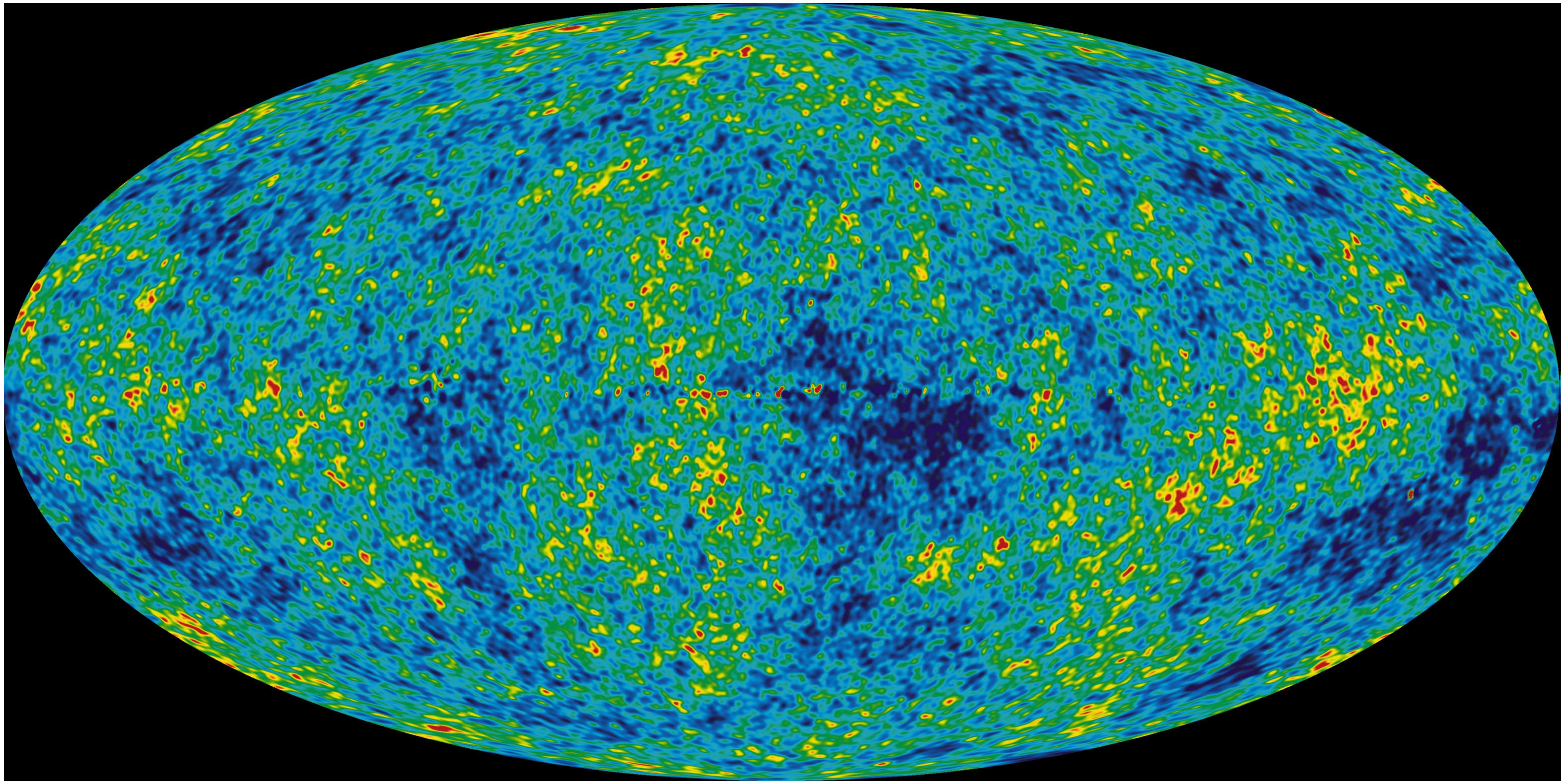
Some Radio Astronomical Sources



Courtesy: NRAO/AUI and N. Bartel, M. Bietenholz, M. Rupen, et al.

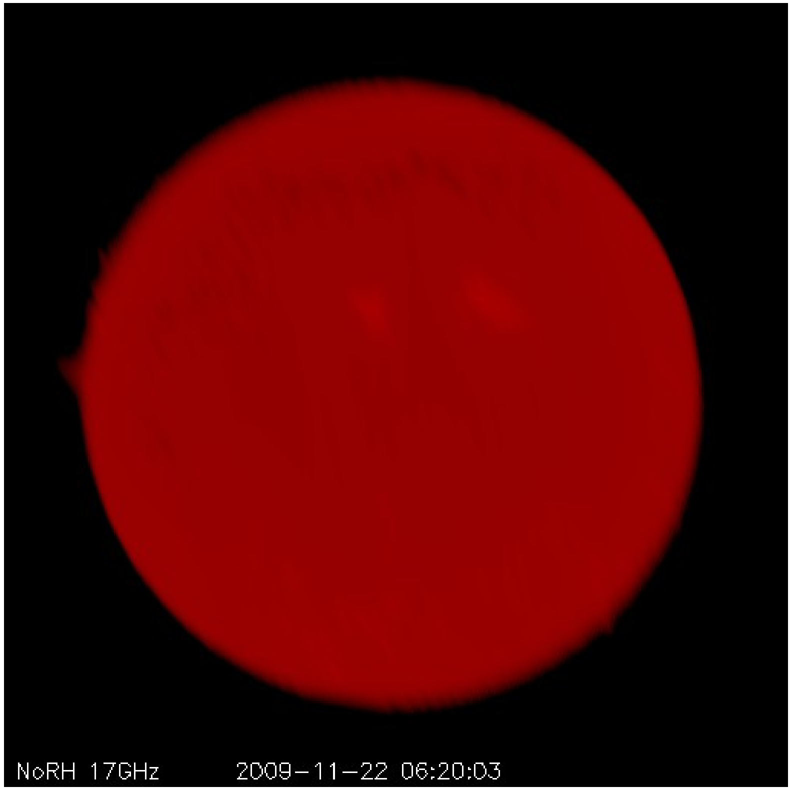
A radio study of super nova SN 1993J, located in M81spiral galaxy at 11 million light-years. Images constructed by Very Long Baseline Interferometry using VLA, and various tracking stations around the world. The sequence of images are constructed by observing at regular intervals. It shows a shell-like radio structure that has expanded for seven years with circular symmetry. The color scale represents the brightness of the radio emission, with blue being faintest and red brightest.

Some Radio Astronomical Sources

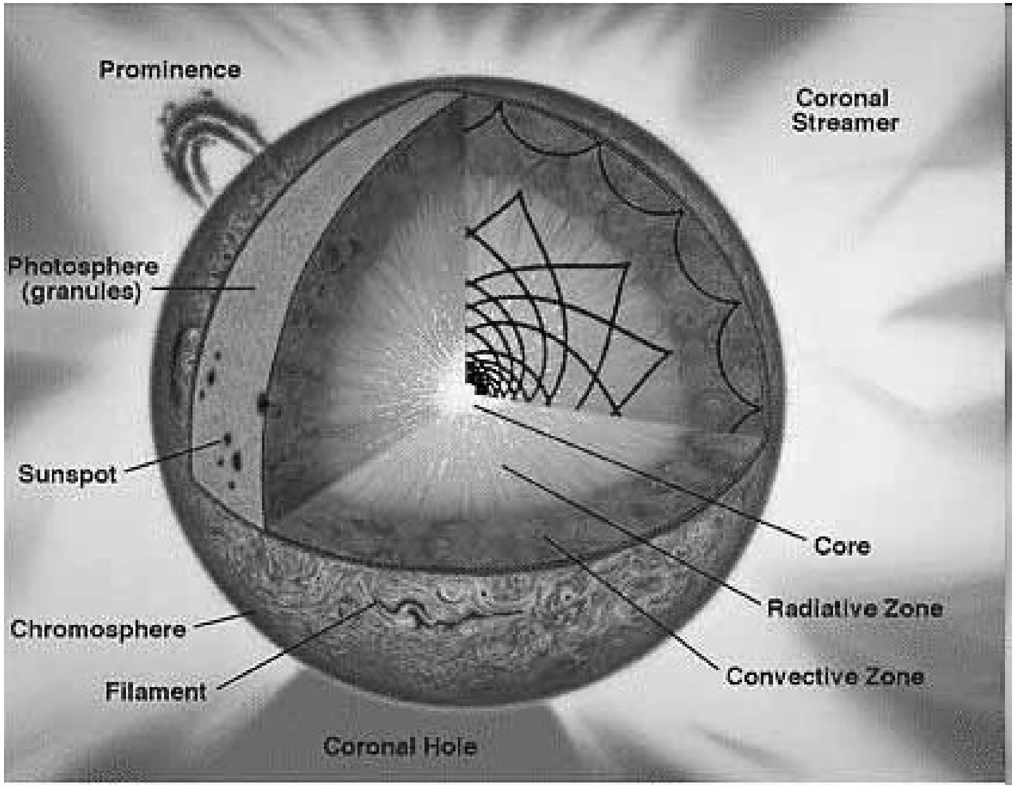


With an optical telescope, the space between stars and galaxies (the *background*) is pitch black. But with a radio telescope, there is a faint background glow, almost exactly the same in all directions, that is not associated with any star, galaxy, or other object. It is cosmic microwave background radiation, and was discovered in 1964 by Arno Penzias and Robert Wilson.

Some Radio Astronomical Sources



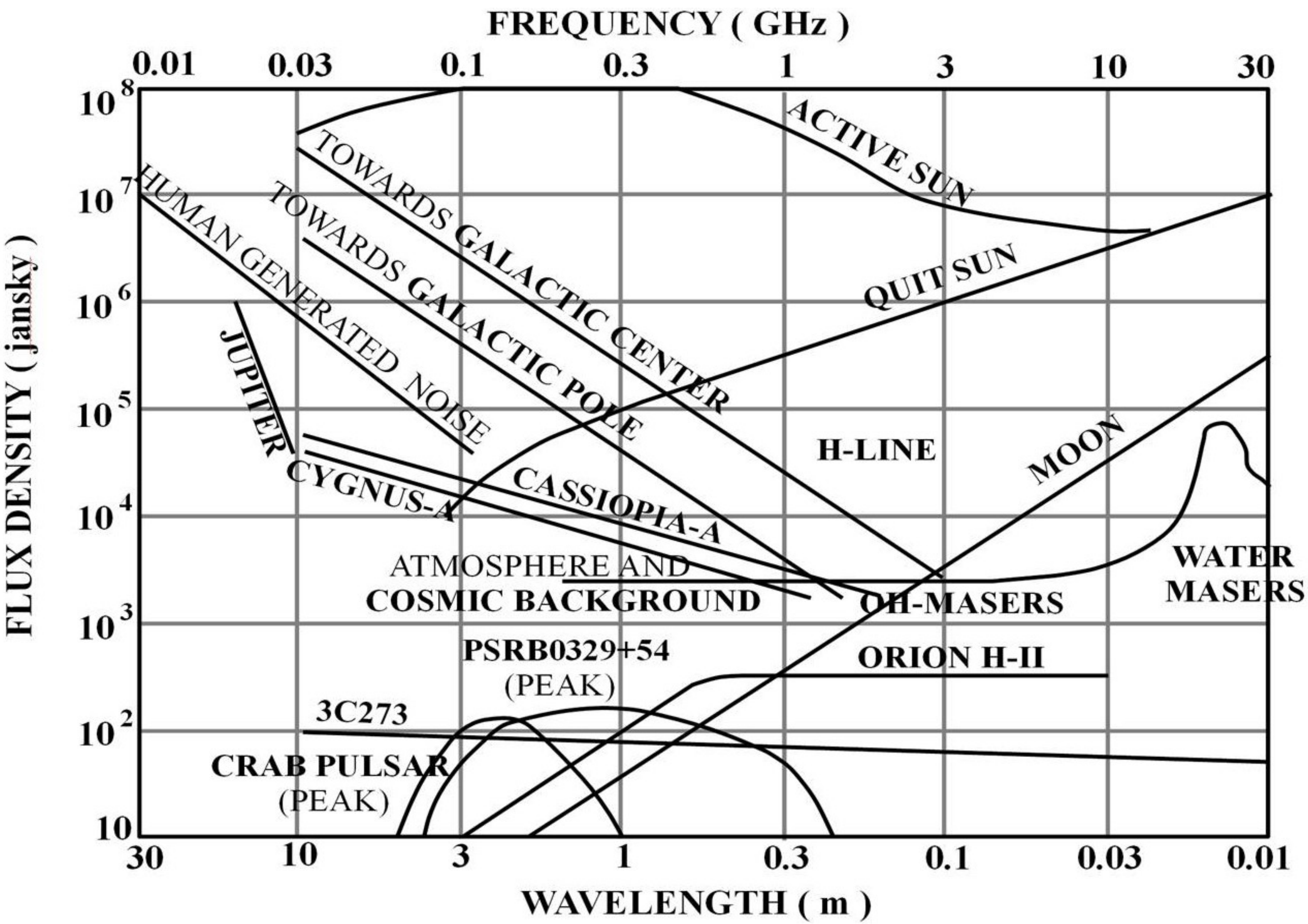
The Sun in Radio (1.7 cm).



Layers of the Sun.

Radio waves penetrate through outer layers of solar gas (**chromosphere** and **corona)**. The penetration depth depends on the wavelength. The left image shows the structure of Sun's atmosphere near the transition region between the chromosphere and the corona, about 2000-2200 km above the photosphere. The right diagram gives the correct picture of the different layers and structure of the Sun.

Radio Flux density on Earth



Radio flux density obtained on Earth resulting from different sources at different frequencies using a 3 sq. m. antenna aperture.

Basic Emission Mechanisms

- **Thermal Radiation**

  Black body radiation with peak emission wavelength dependent on the temperature (obeys Planck Law).

- **Non-Thermal Radiations:-**

  (*i*) ***Synchrotron Radiation*****:** From motion of charged particles trapped in magnetic fields.

  (*b*) ***Bremsstrahlung Radiation*****:** From acceleration and deceleration of charged particles.

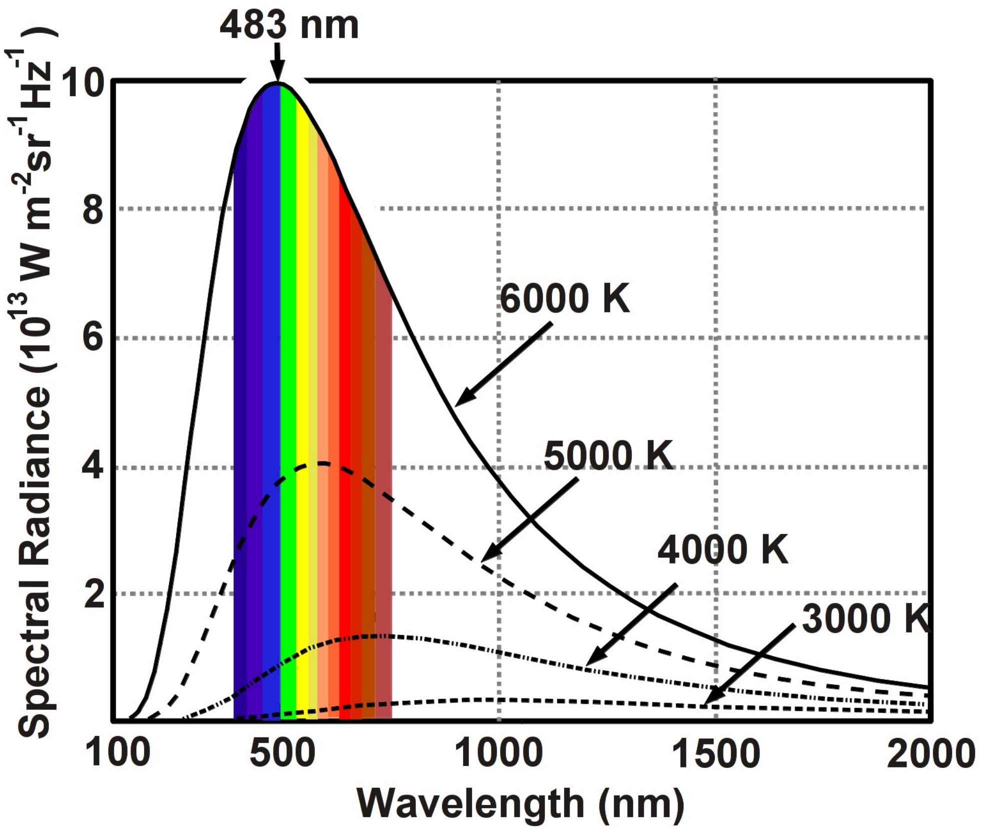
Basic Emission Mechanisms (Thermal)

Planck's Law

Spectral Radiance (brightness *B*)

$$B = \frac{2h\nu}{\lambda^2}\frac{1}{e^{\frac{h\nu}{kT}}-1}$$

W m^{-2} sr^{-1} Hz^{-1}



Basic Emission Mechanisms (Thermal)

Steffan-Boltzmann law

$\left(\sigma = 5.7 \times 10^{-8}\ \mathrm{W\,m^{-2}} K^{-4}\right)$

$$F = \sigma T^4\ \mathrm{W/m^2}$$

Luminosity L of star

$$L = 4\pi \mathrm{R}^2 \sigma T^4\ W$$

Wien's radiation law
(brightness B)

$$B = \frac{2h\nu}{\lambda^2} e^{-\frac{h\nu}{kT}}\ \mathrm{W/m^2/str/Hz}$$

Wien's displacement law

$$\lambda_{peak} = \frac{2.9 \times 10^{-3}}{T} m$$

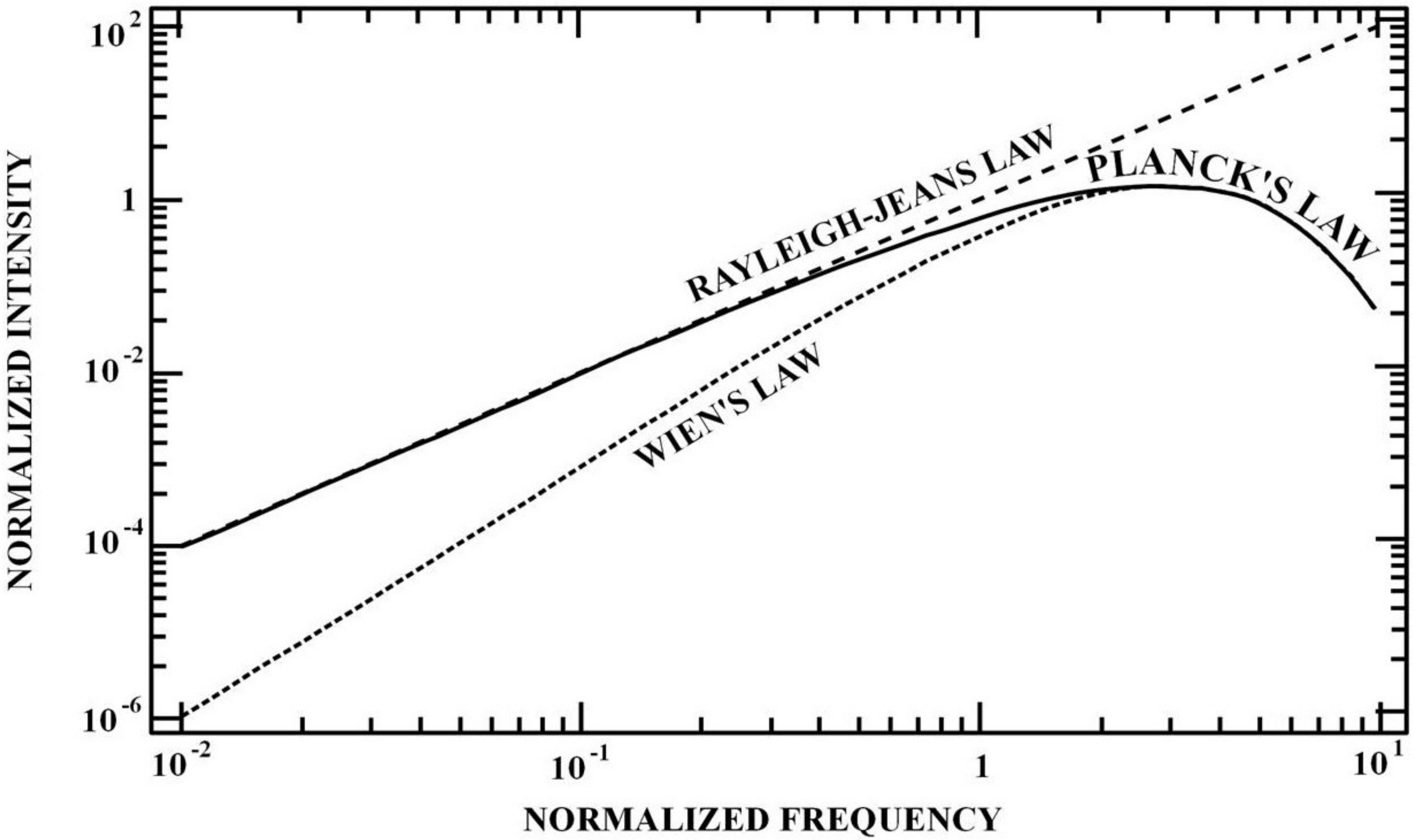
Rayleigh-Jeans law
(brightness B)

$$B = \frac{2kT}{\lambda^2}\ \mathrm{W/m^2/str/Hz}$$

Apparent brightness
(observed flux f)

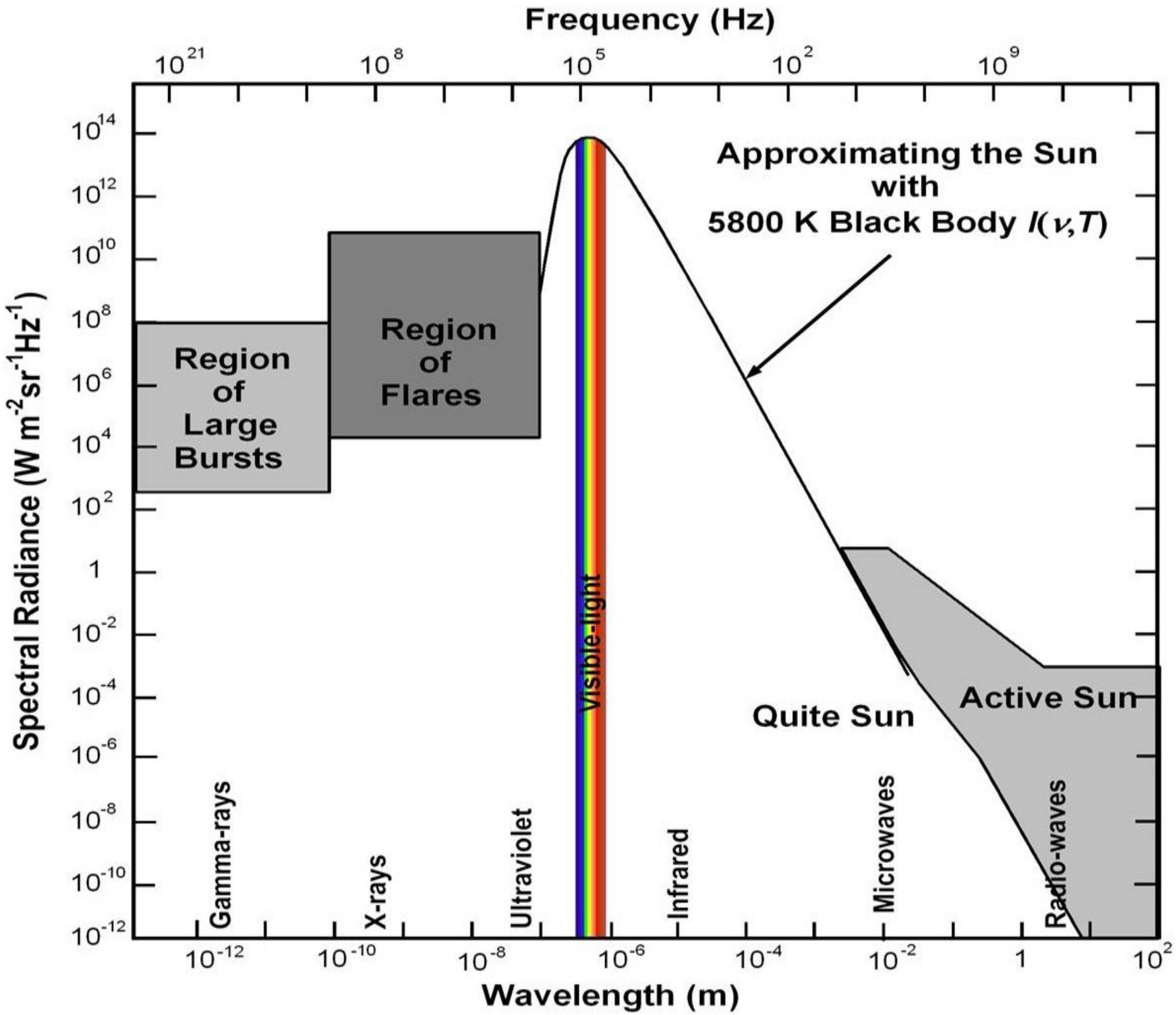
$$f = \frac{L}{4\pi r^2}\quad \mathrm{W/m^2}$$

Basic Emission Mechanisms (Thermal)



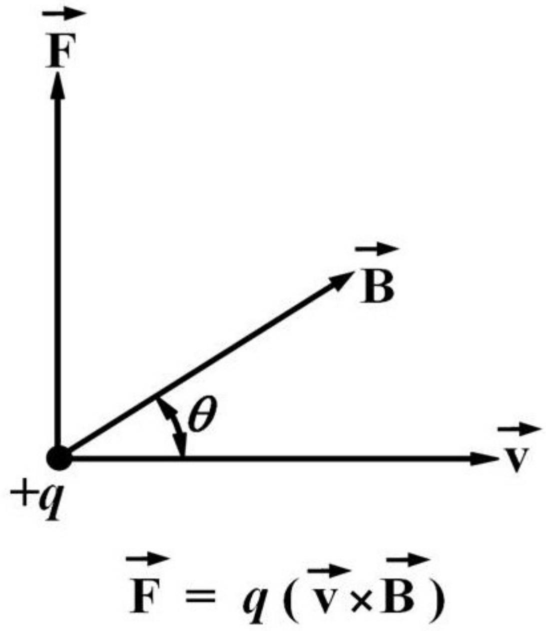
Comparison of Rayleigh-Jean law and Wien’s law with Planck’s law.

Basic Emission Mechanisms (Thermal)



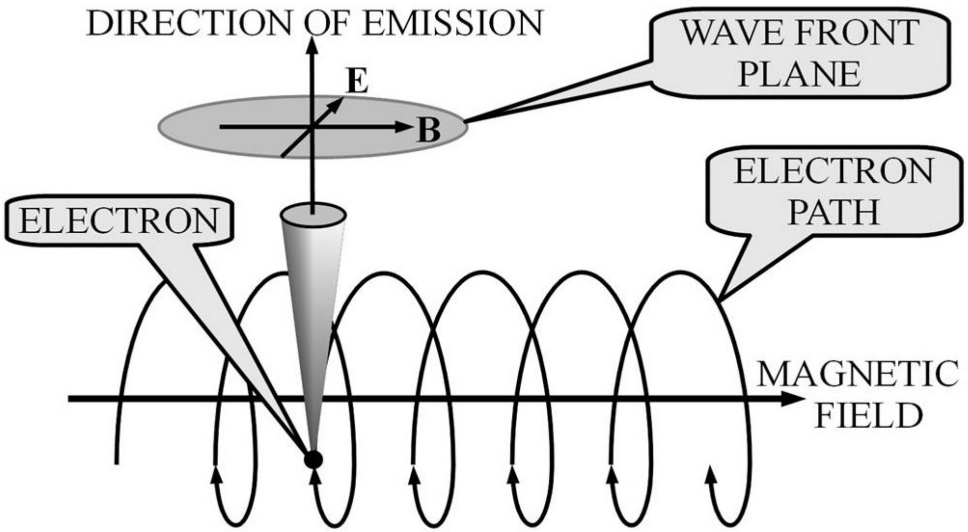
The Sun is approximated as black body at 5800 K.
(Reasonable range 10^{-7} to 10^{-3} meters.)

Basic Emission Mechanisms (Synchrotron)

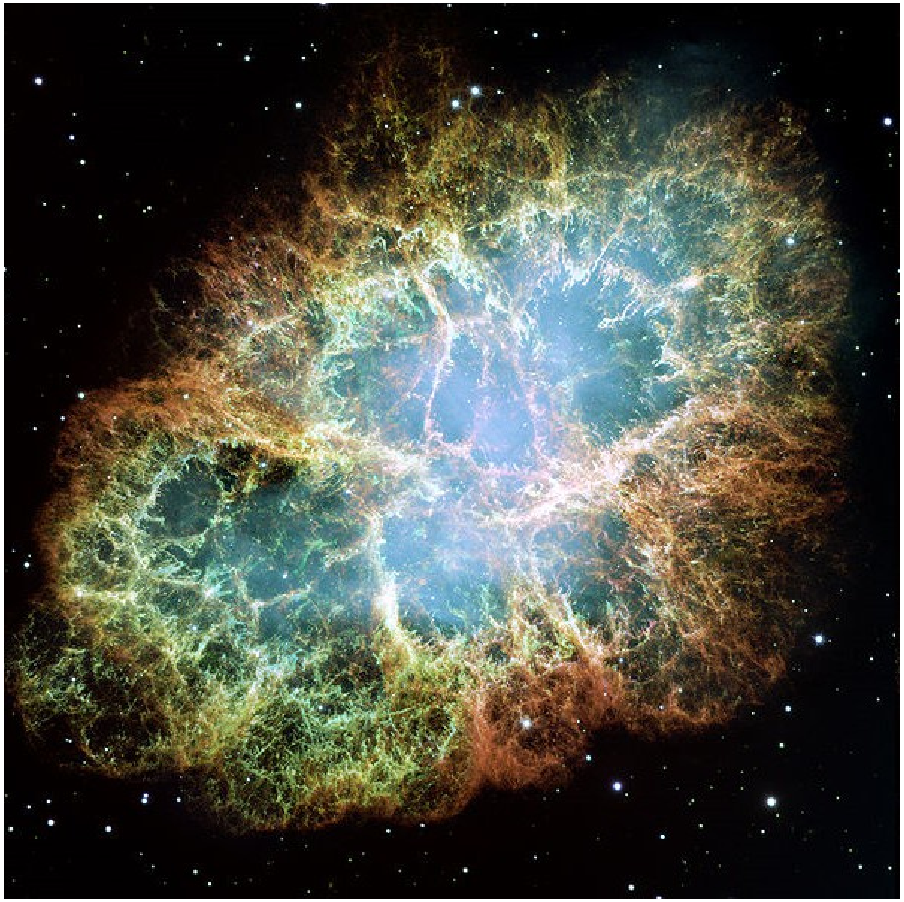


Force on a moving charge in a magnetic field = cross product of particle velocity with magnetic field times magnitude of charge.

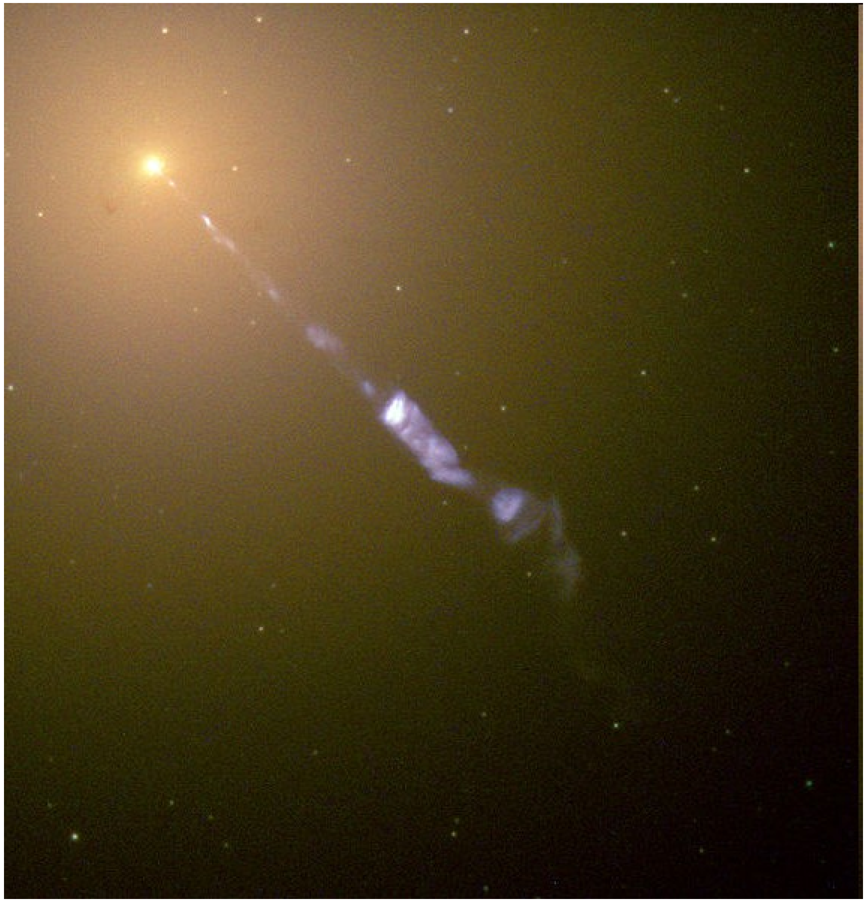
An electron at relativistic speed spirals around a magnetic field line resulting to synchrotron emission. It is polarized with electric field parallel to the shortest distance between the electron and magnetic field.



Basic Emission Mechanisms (Synchrotron)



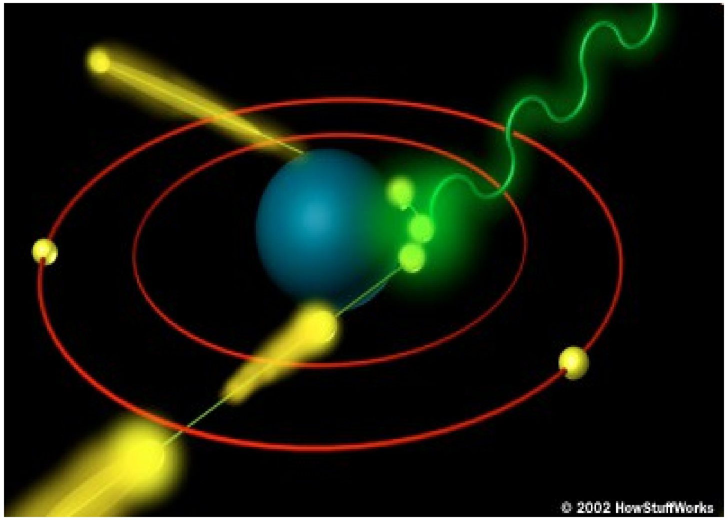
The bluish glow from the central region of the Crab Nebula is due to synchrotron radiation.



M87's Energetic Jet., HST image. The blue light from the jet emerging from the bright AGN core, towards the lower right, is due to synchrotron radiation.

Basic Emission Mechanisms (Bremsstrahlung)

It is the electromagnetic radiation produced by acceleration of any charged particle when deflected by any other charged particle. It has a continuous spectrum. It is continuously happening in a plasma from mechanical interaction of free electrons with ions.

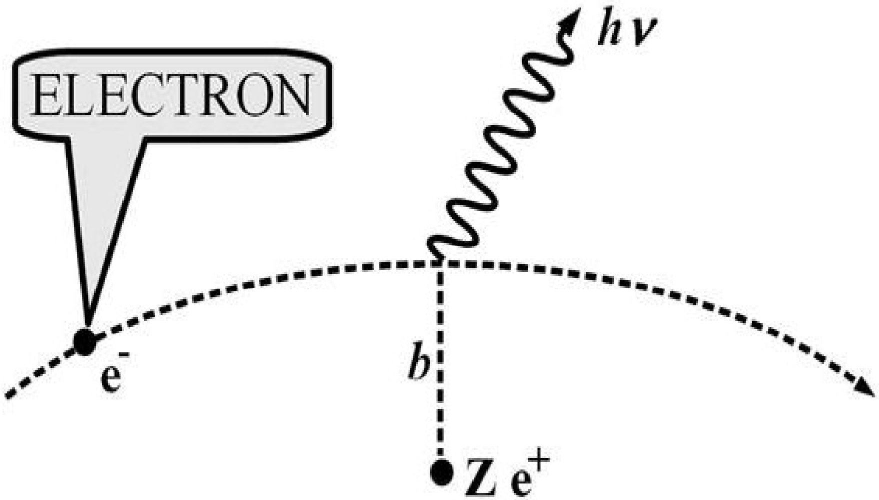


In a uniform plasma, with thermal electrons the power spectral density of the Bremsstrahlung radiated is given as

$$\frac{dP_{\mathrm{Br}}}{d\omega} = \frac{4\sqrt{2}}{3\sqrt{\pi}} \left[n_e^2 r_e^3\right] \left[\frac{(m_e c^2)^{3/2}}{(k_B T_e)^{1/2}}\right] Z_{\mathrm{eff}} E_1(w_m), \qquad w_m = \frac{\omega^2 m_e}{2 k_m^2 k_B T_e}$$

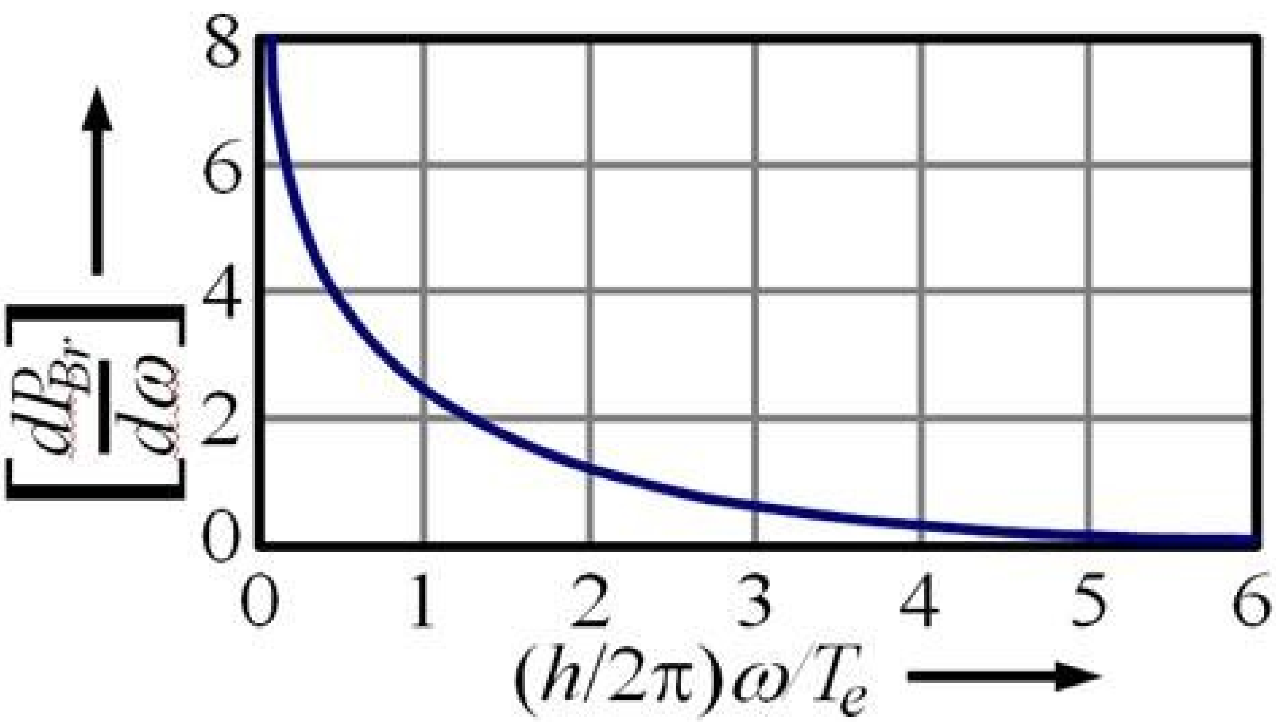
T_e = Electron's temperature, n_e = Electron's number density, m_e = Electron's mass, r_e = Electron's classical radius, k_B = Boltzmann constant, c = speed of light, Z_{eff} = Effective state of ion charge, $E_1(w_m)$ is exponential integral and k_m is a maximum or cutoff wavenumber

Basic Emission Mechanisms (Bremsstrahlung)

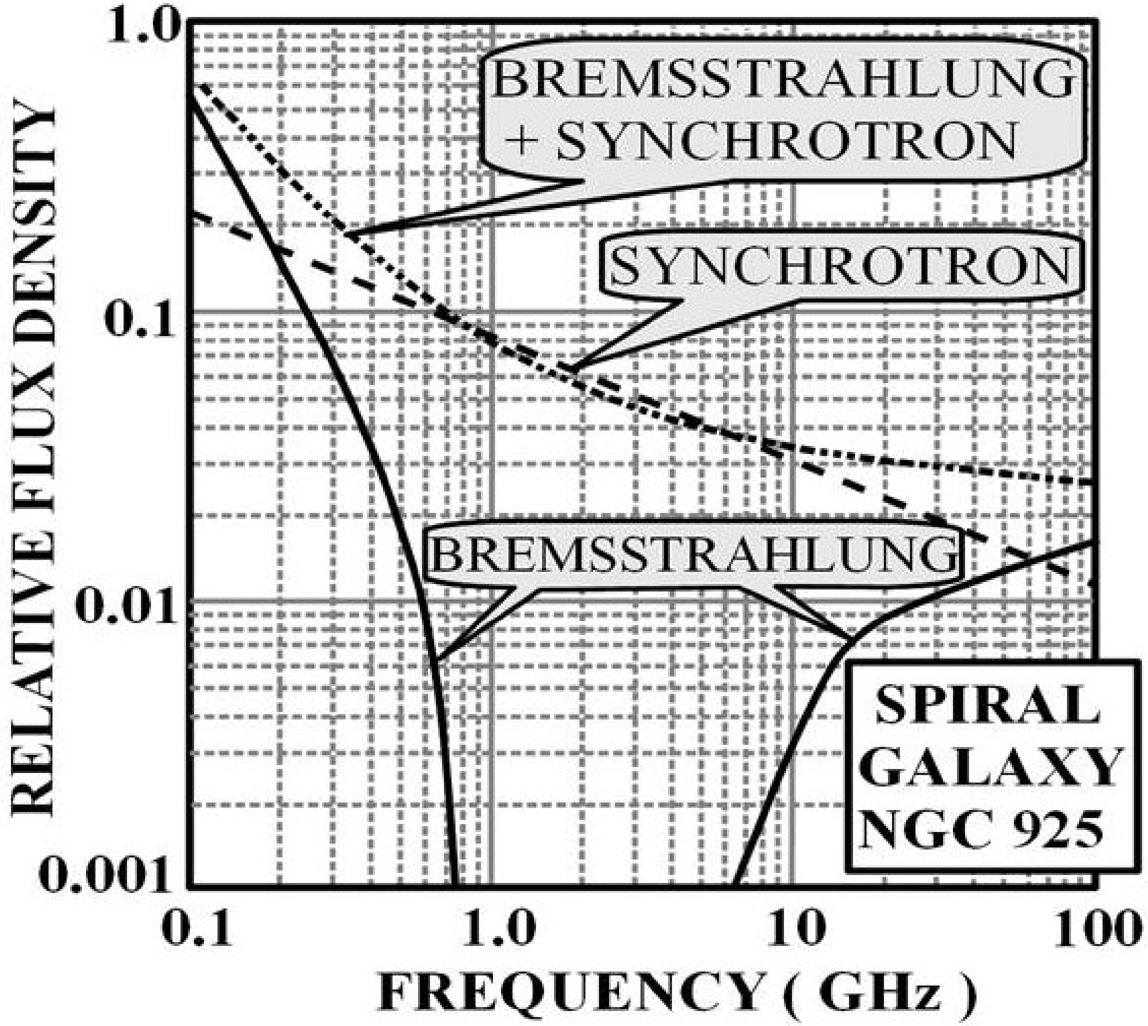


An electron $e-$ crossing an ion with charge $Z\,e+$ undergoes non-uniform acceleration/decelaration resulting Bremsstrahlung emission of photons $h\,v$.

A tentative plot of Bremsstrahlung spectrum using arbitrary units.

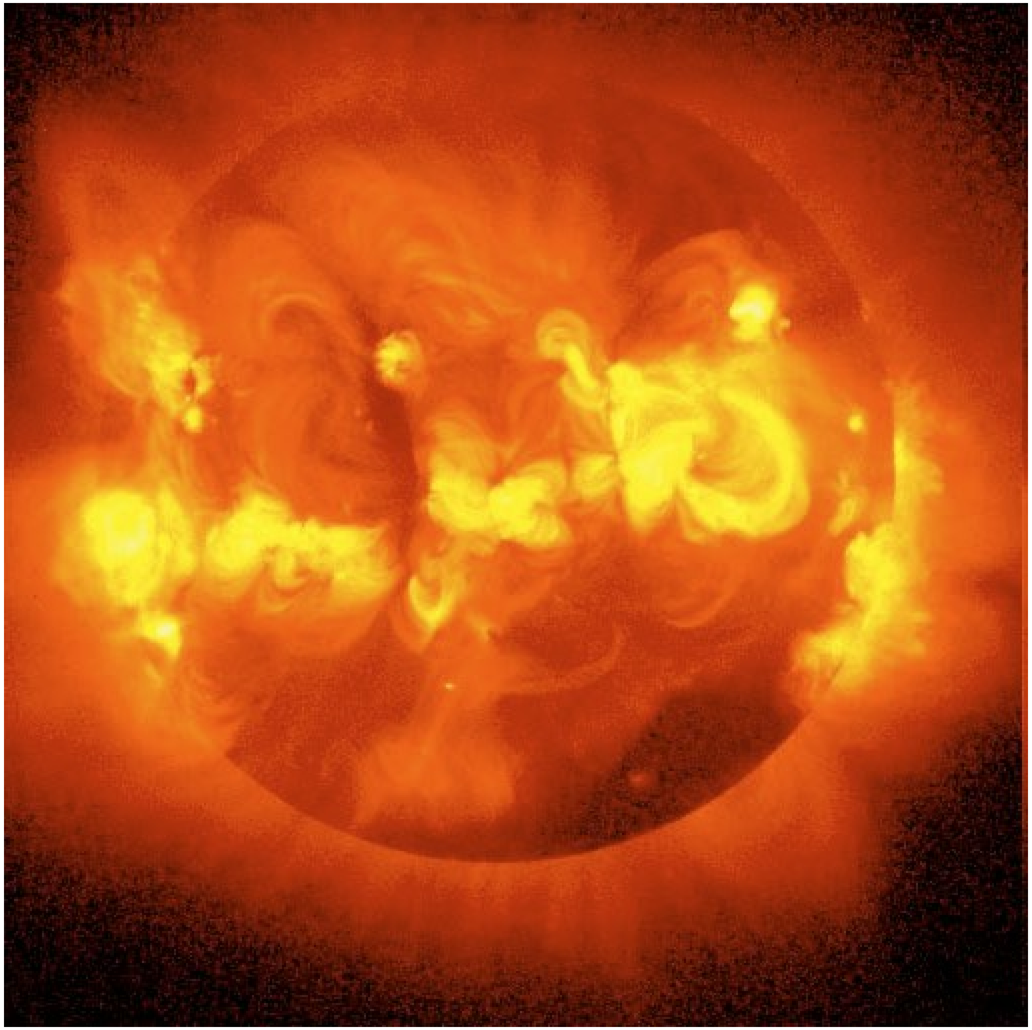


Basic Emission Mechanisms (Bremsstrahlung)



Bremsstrahlung and Synchrotron radiations from the spiral galaxy NGC925.

Basic Emission Mechanisms (Bremsstrahlung)



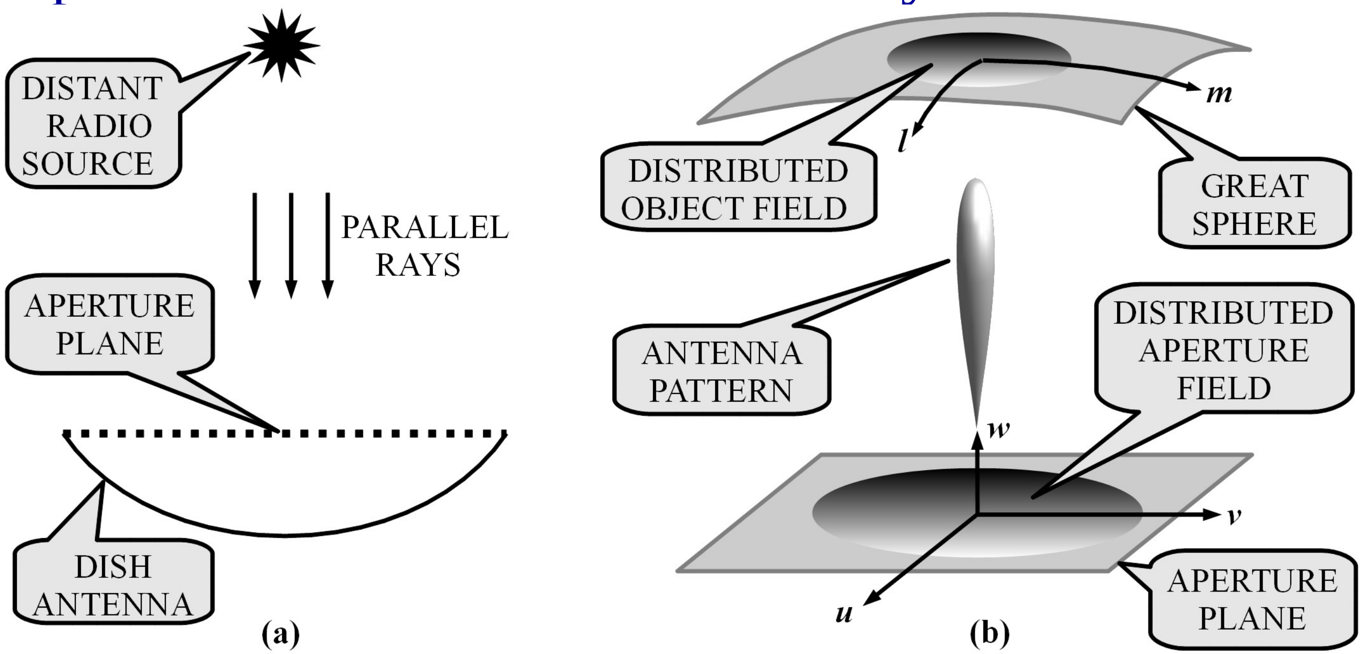
The picture is a soft X-ray thermal bremsstrahlung coronal image of the Sun, taken by the ***Yohkoh Solar Observatory*** on February 1st, 1992. It highlights the magnetic field loop structure that forms the precursor environment to solar flare activity.

Principles of Aperture Synthesis

Basic Mapping Technique

Assume a single dish telescope. Its aperture is at the origin of (u,v,w) coordinates and w points to the source. Parallel rays illuminate the aperture.

On the sky let the two dimensional electric field distribution be centered on (l,m) coordinates.

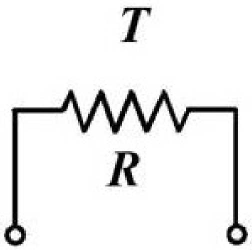


On $u-v$ plane, the electric field distribution $E(u, v)$ is a result of the electric field distribution $V(l,m)$ on the sky. $W(u,v)$ is auto-correlation of each electric field points on $u-v$ plane.

Mapping: *A two dimensional Fourier transform of* $W(u, v)$ *is the intensity field distribution* $I(l,m)$ *of the source in* $l-m$ *space.*

Receiving Principles: Antenna Temperature

Consider a resistor R at temperature T. If $\Delta\nu$ is the bandwidth in Hz, then power W available at its terminals (in watts) is given as:



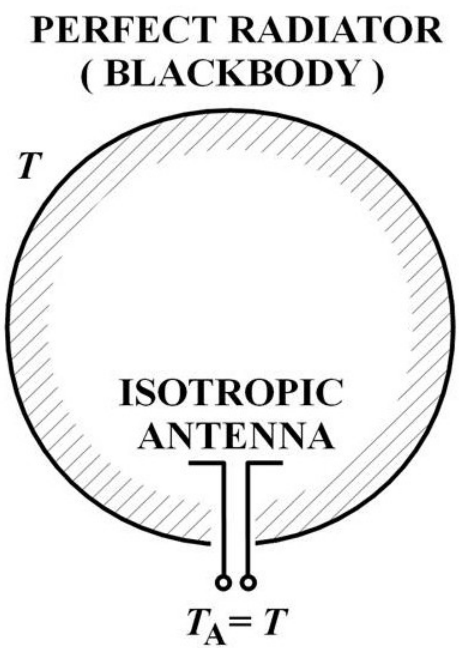
$$W = kT\Delta\nu$$ where, $k = 1.38\text{x}10^{-23}$ J/K (Boltzmann const.)

The spectral power w per unit bandwidth in watts/Hz is $w = kT$

Let us take a lossless matched isotropic antenna having a radiation resistance $R_r = R$. The power at it terminals will be:

(*i*) Zero if the antenna doesn't receive any radiation.

(*ii*) Greater than zero if radiation is received.



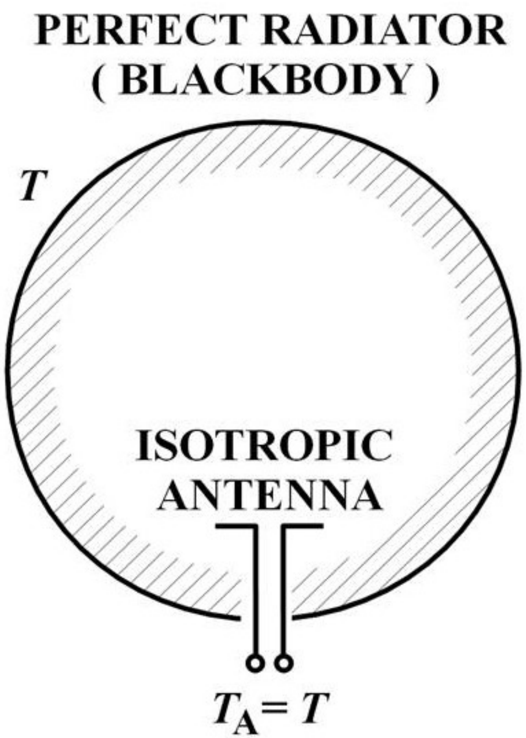
If we place this antenna inside a black body at a temperature T, so that it collects only the radiation and converts into spectral power w_A, we find

$$w_A = w \quad \text{or, } kT_A = kT \quad \text{i.e., } T_A = T$$

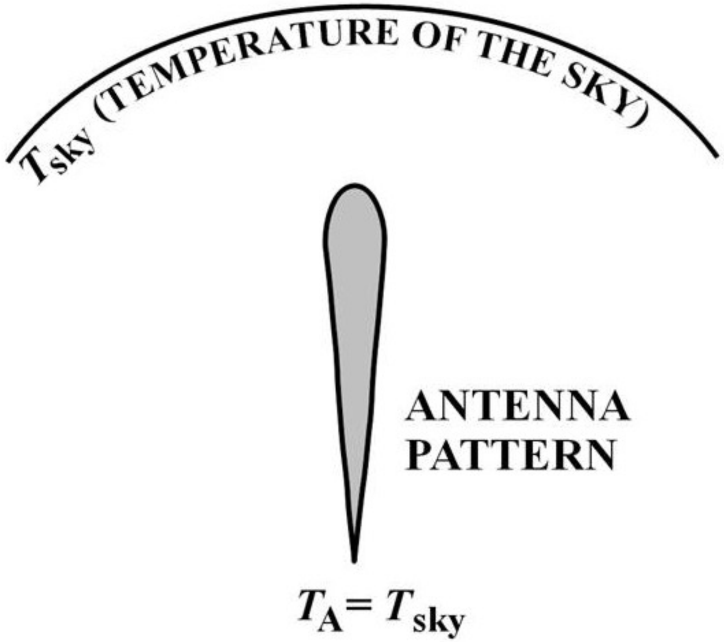
where, T_A is known as the ***antenna temperature***.

Receiving Principles: Antenna Temperature

Antenna Temperature T_A describes how much noise an antenna produces in a given environment. It is not the physical temperature of the antenna.

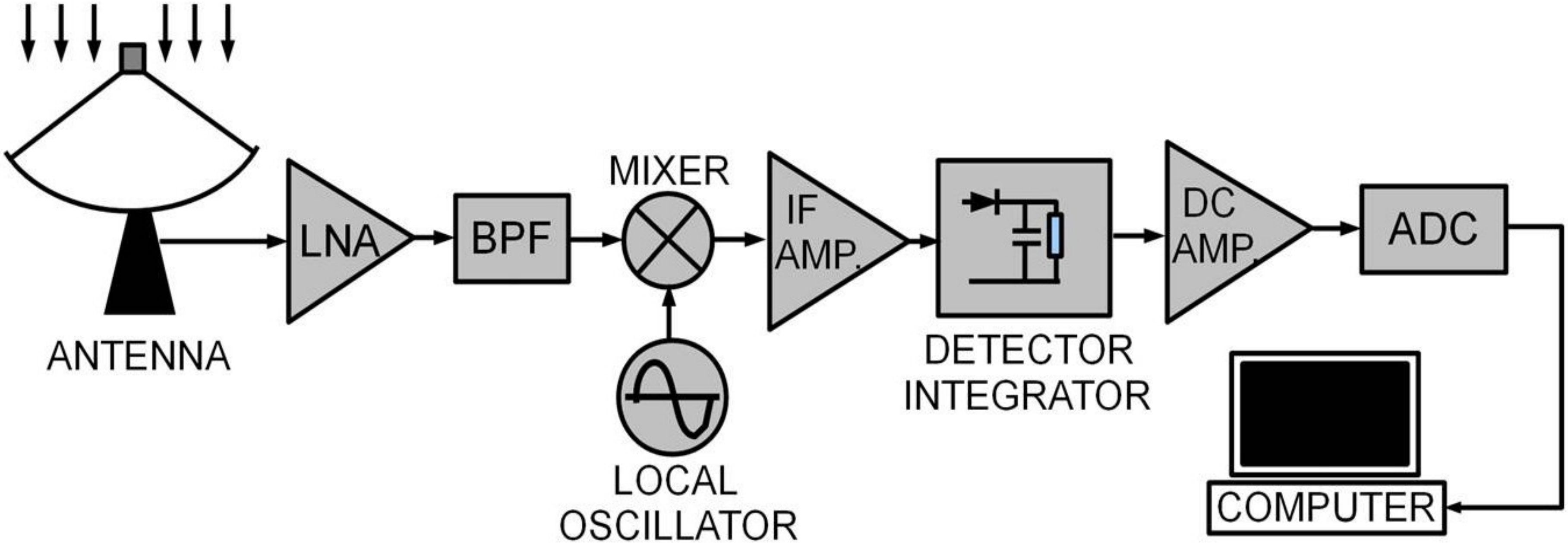


An isotropic antenna enclosed inside a black body enclosure at a temperature T. The antenna temperature T_A is equal to the black body temperature T.



Sky brightness temperature T_{sky} is picked by an antenna since its radiation pattern is pointing towards it.

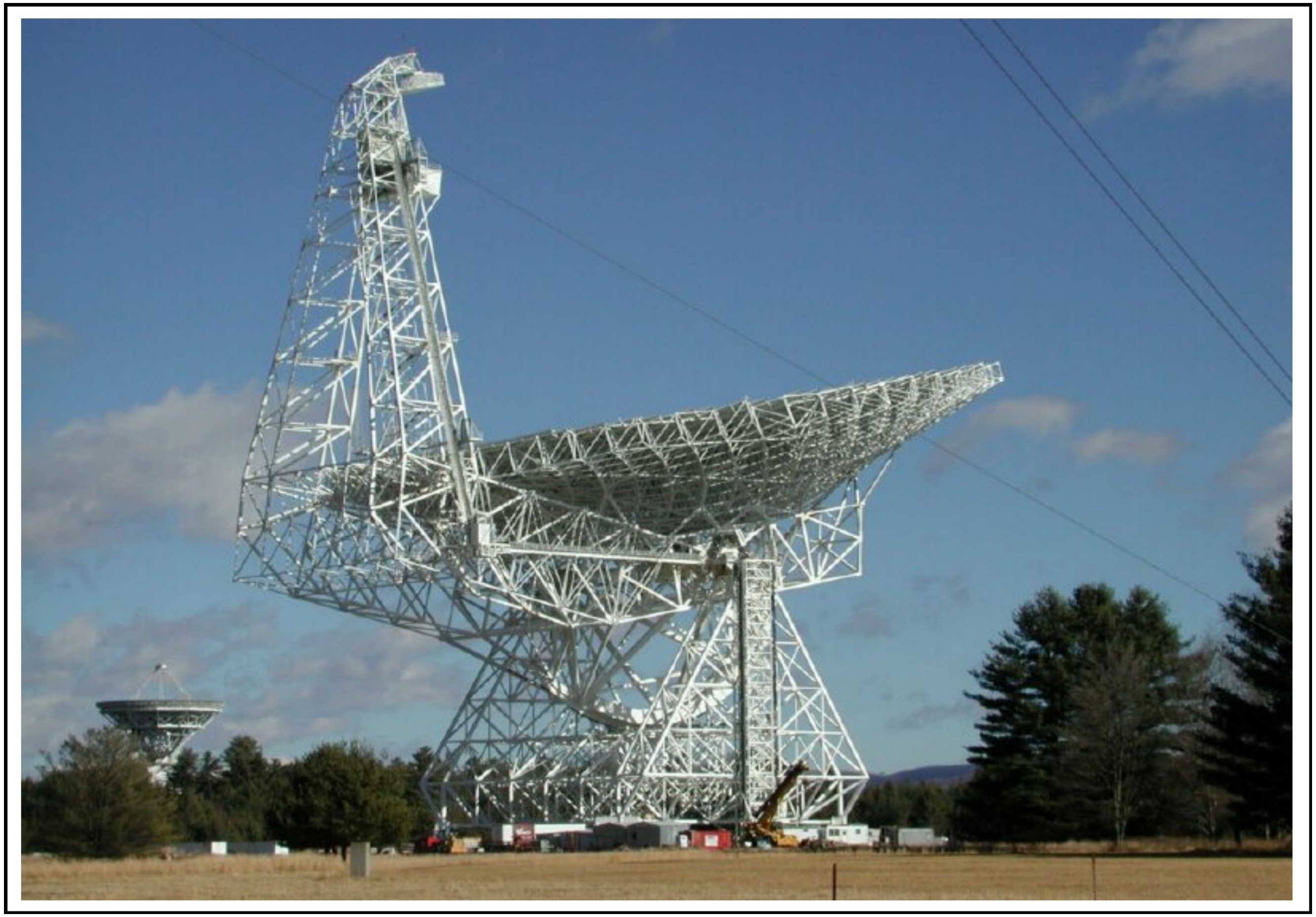
Receiving Principles
(Simple Heterodyne Radio Telescope)



A basic heterodyne radio telescope receiver. The antenna, low noise RF amplifier (LNA), mixer, local oscillator, IF amplifier, detector, DC amplifier and data recording computer are shown.

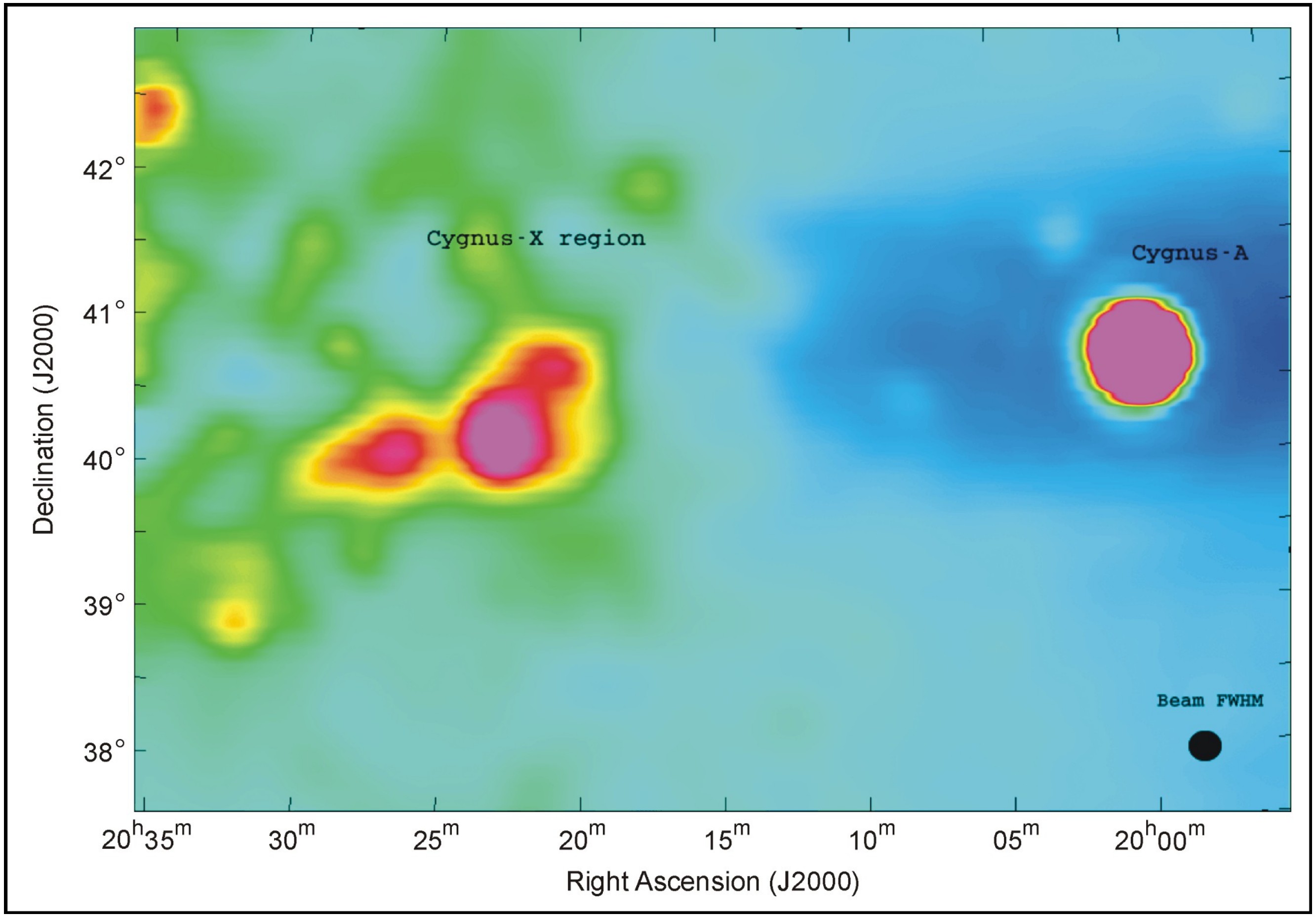
Single Dish Radio Telescope

Single dish radio telescopes are very few mainly due to the limitation of large dish construction.



World's largest steerable single dish radio telescope named as the Green Bank Telescope (GBT) located at NRAO, West Virginia.

Single Dish Radio Image



An image of the Cygnus-X region at 790 MHz in Equatorial coordinates. It was made during the commissioning of GBT. The beam-width is approximately 16 arc-min. The signals from Cygnus-A is completely saturated. Copyright NRAO/AUI/NSF.

Receiving Principles
(Techniques for improving Signal to Noise Ratio)

Consider two signals consisting of a common signal and different noise signals as functions of time. If the signals are multiplied, the common signals boosts up whereas the noise part diminishes. If an integration is performed over time, the signal to noise ratio further improves.

Cross-correlation

For two continuous functions of time *t* denoted as *f*(*t*) and *g*(*t*), the cross-correlation *r* is given as:

$$r(\tau) = \int_{-\infty}^{\infty} f^*(\tau)\ g(t+\tau)\, d\tau \quad \text{and} \quad r(m) = \sum_{n=-\infty}^{\infty} f^*(n)\ g(n+m)$$

Auto-correlation

A special case of cross-correlation is auto-correlation where the signal is correlated with itself. Auto-correlation $R(\tau)$ of a continuous function *f*(*t*) of time *t* is given as

$$R(\tau) = \int_{-\infty}^{\infty} f^*(\tau)\ f(t+\tau)\, d\tau \quad \text{and} \quad R(m) = \sum_{n=-\infty}^{\infty} f^*(n)\ f(n+m)$$

Receiving Principles
(Techniques for improving Signal to Noise Ratio)

Wiener-Khinchin Theorem

It is also known as the *Wiener-Khintchine* theorem. It states that ***the power spectral density of a wide sense stationary random process is the Fourier transform of the corresponding autocorrelation function***. Mathematically, if $S(v)$ is the power spectral density of a continuous random signal $x(t)$, then it may be expressed as the Fourier transform of the auto-correlation function $R(\tau)$.

$$S(\nu) = \int_{-\infty}^{\infty} R(\tau)\, e^{-j2\pi\nu\tau}\, d\tau$$

Radio Arrays

Generally two types of radio arrays are used in radio astronomy: (*i*) phased array, and (*ii*) correlator array.

Advantages of arrays

- Difficulties in constructing a large single dish can be overcome by arrays.
- Multiple increase in signal to noise ratio as compared to a single dish.
- Increases overall directivity and resolution of the telescope.
- Effect of Earth's rotation can aid to super-synthesis.

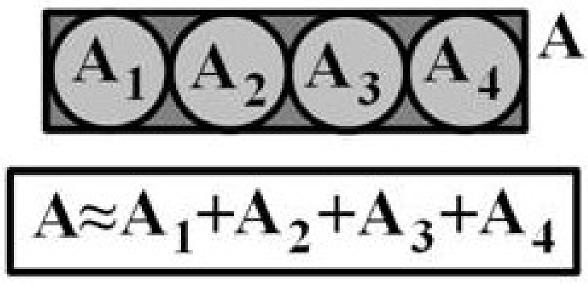
Note: The GMRT configures itself as correlator array when observing spectral lines and continuum radio sources. It configures itself as a phased array when observing pulsars.

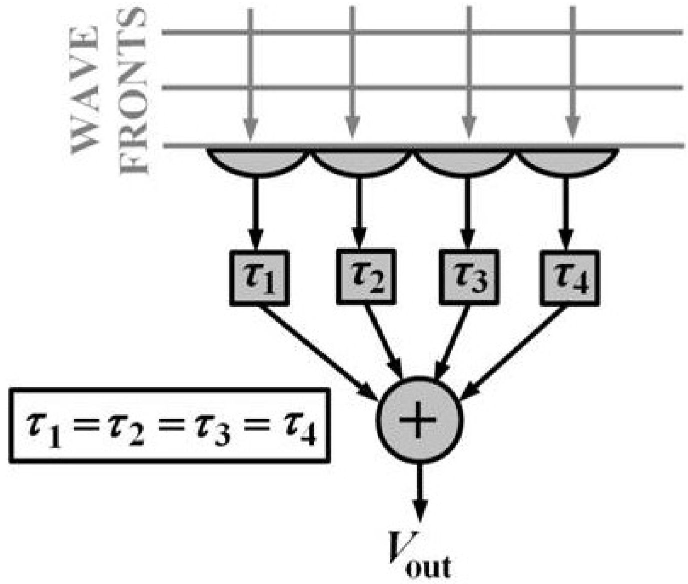
Radio Arrays (Filled Phased Array)

Synthesizing a large aperture using small apertures.

Synthesis of a rectangular aperture with four circular apertures.

$$A \approx \sum_{1}^{4} A_i$$





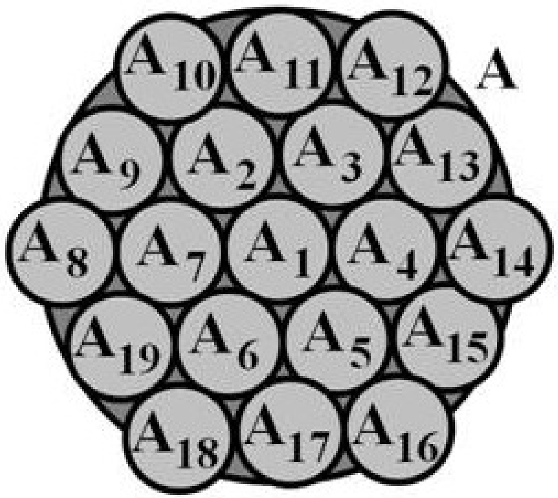
At any instant of time, the same wave-front reaches all the individual apertures.

$$\tau_1 = \tau_2 = \tau_3 = \ldots$$

<u>Disadvantage</u>: The plane of the array must be parallel to the wave-fronts. Not suitable for a moving source on the sky.

Radio Arrays (Filled Phased Array)

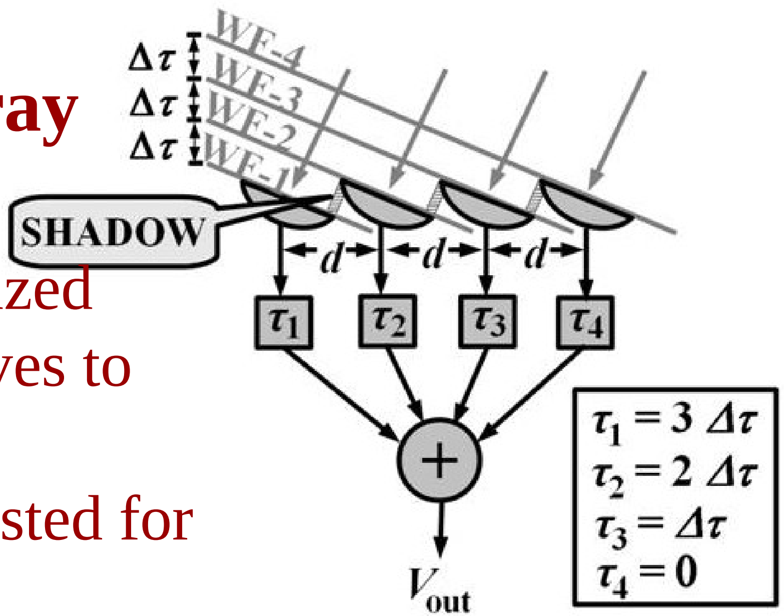
Synthesizing a large aperture using small apertures



Synthesis of a large circular aperture using many circular apertures.

$$A \approx \sum_{1}^{19} A_i$$

A tracking antenna array



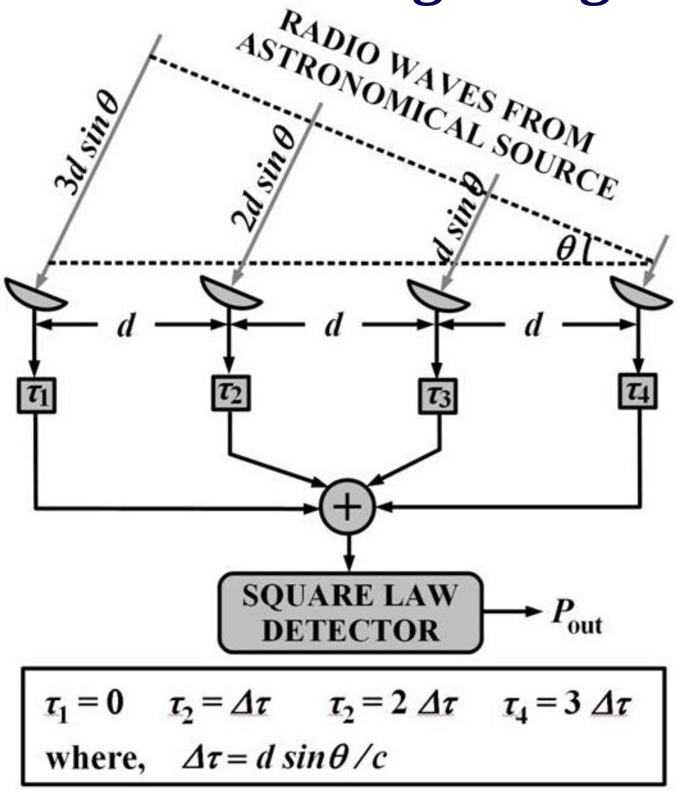
<u>Disadvantages</u>

(i) The effective area of the synthesized antenna decreases as the source moves to the horizon.

(ii) The delay has to constantly adjusted for a moving source

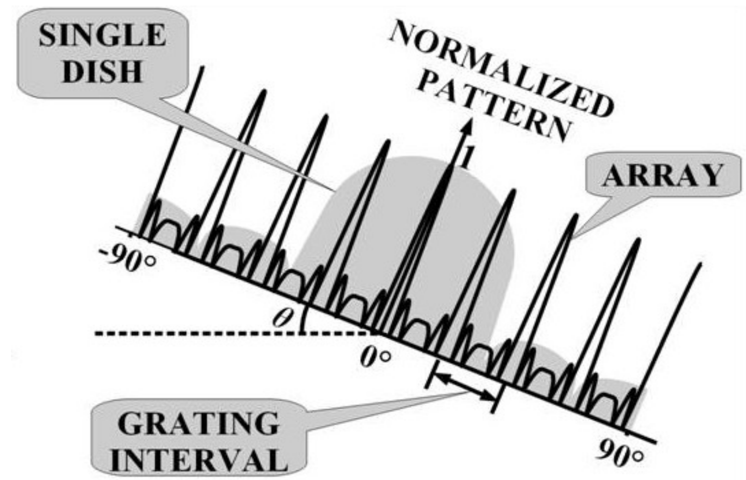
Radio Arrays (Grating Phased Array)

A grating array and its response

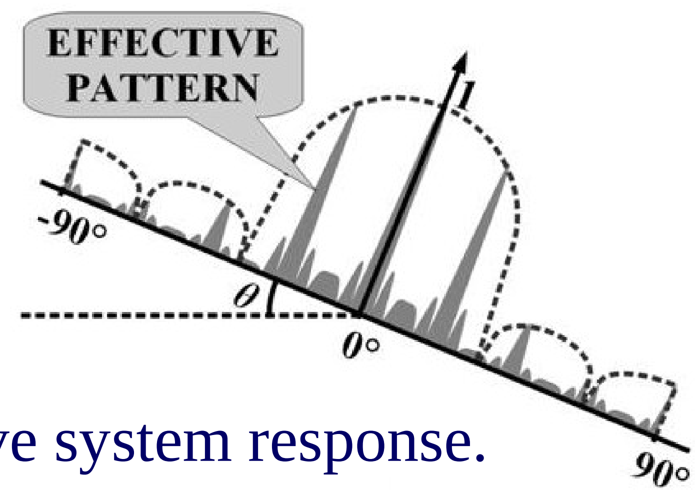


$$P_{out} = \left(\sum_{1}^{n} V_i \right)^2$$

Signals are added after phase correction using appropriate delays.

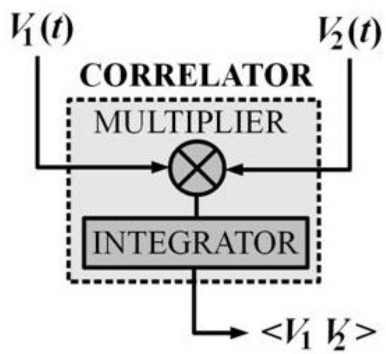


Normalized power patterns of a dish and an array of isotropic antennas.



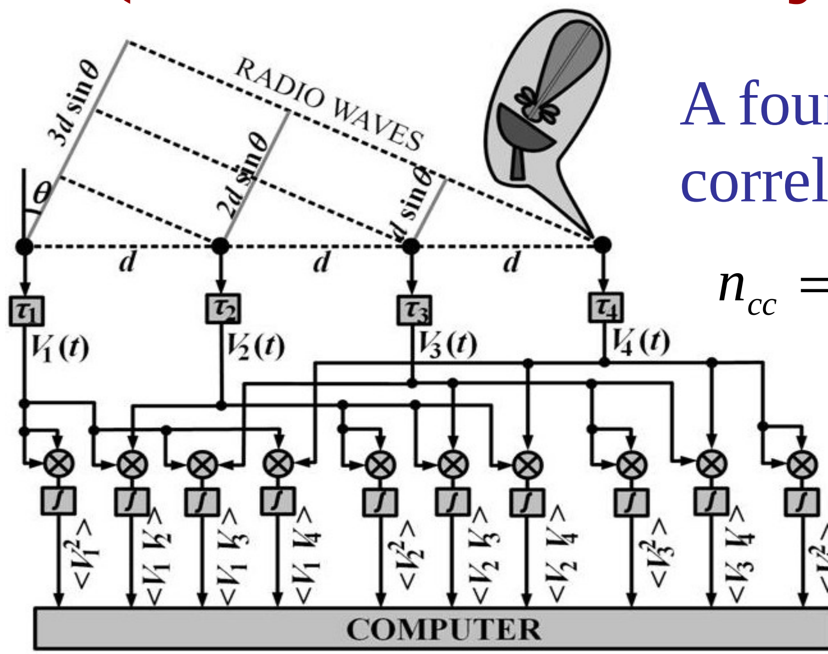
Effective system response.

Radio Arrays (Correlator Arrays)



$$r(\tau_g) = V_1 V_2 \cos\left(2\pi\nu\tau_g\right)$$

Basic correlator unit: A multiplier followed by an integrator.



A four antenna correlator array.

$$n_{cc} = n_a\left(n_a - 1\right)/2$$

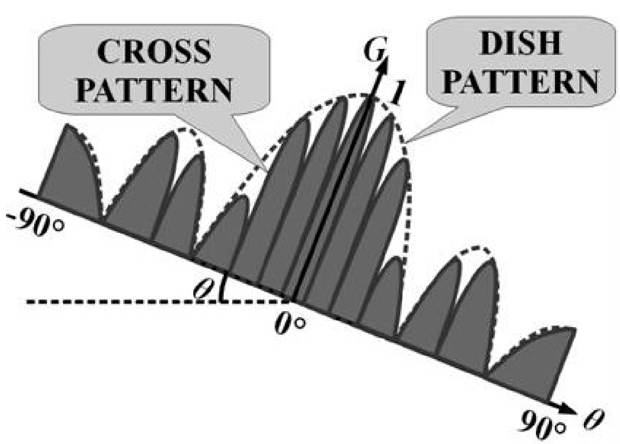
$\tau_1 = 3\, d \sin\theta / c$

$\tau_2 = 2\, d \sin\theta / c$

$\tau_3 = d \sin\theta / c$

$\tau_4 = 0$

Signals from each antenna pair are cross-correlated and saved in a computer along with autocorrelation products.



Tentative plot of the fringe pattern formed by an antenna pair in cross-correlation.

Introduction to Radio Frequency Interference (RFI)

Signals of radio astronomy are extremely weak with respect to the man made radio signals noise. For instance, the radio flux received from our nearest star Sun could be only a few SFU[1] (solar flux unit), whereas a distant radio station may produce several millions of SFUs. The power flux densities received from distant radio galaxies varies from micro-jansky to milli-jansky. We have seen that astronomical sources produce signals like a Gaussian noise, generally in both polarizations over a wide frequency range. A locally generated wideband radio noise like that from an electrical arc welding may be quite high to obscure the astronomical signal. A noise or a signal of any kind interfering or obscuring the astronomical signals is termed as RFI by radio astronomers.

[1] 1 SFU = 10^{-22} $\mathrm{Wm^{-2}\,Hz^{-1}}$ = 10^{4} jansky.
1 jansky = 10^{-26} $\mathrm{W\,m^{-2}\,Hz^{-1}}$

RFI coupling in Meter-waves

RFI couples with the radio astronomical data through several ways:

(*i*) Side-lobes of the telescope-antenna.

(*ii*) Mesh leakage of the dishes.

(*iii*) Spill over of the feed pattern over the dish.

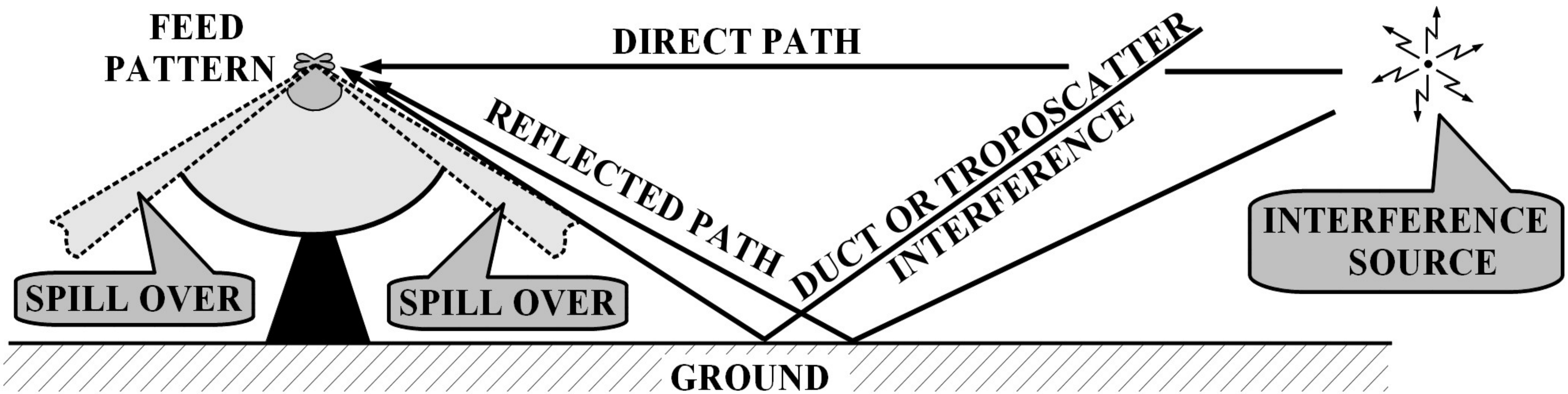
(*iv*) It may also leak into the system by penetrating the shielding of cables and electronic receiver system.

(*v*) Can also be produced from non-linear operation of semi-conductors.

To avoid human generated radio noise, radio telescopes are commissioned at very remote places.

RFI coupling in Meter-wave Antennas

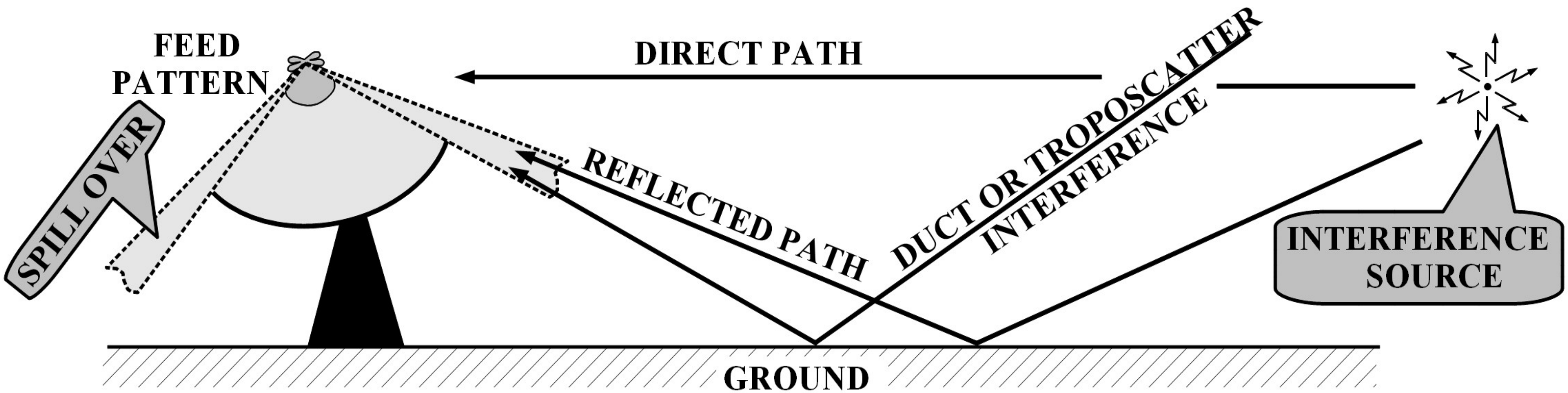
Interfering radio signals entering through telescope antenna in the VHF/UHF frequency bands.



While observing an astronomical source at the zenith, the interfering signals may enter through the side-lobes of the antenna.

RFI coupling in Meter-wave Antennas

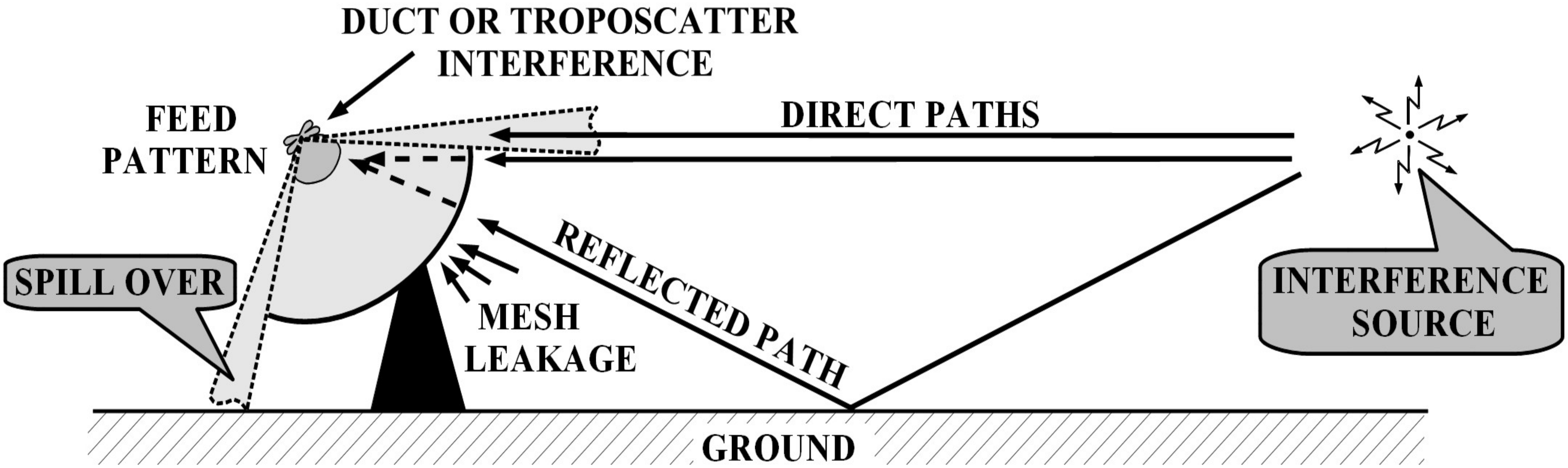
Interfering radio signals entering through telescope antenna in the VHF/UHF frequency bands.



While observing a radio source at a small angle from zenith, interfering signal may enter through the spill-over and side lobes.

RFI coupling in Meter-wave Antennas

Interfering radio signals entering through telescope antenna in the VHF/UHF frequency bands.



While observing a radio source at a large angle from zenith, interfering signals may enter through spill-over, mesh leakage and side lobes.

Principles of Aperture Synthesis

The Basis of Super-synthesis Technique

Instead of a single antenna, an antenna array aided with the rotation of Earth can synthesize a large antenna aperture. This overcomes the difficulty of constructing single large antenna. This technique was named by Martin Rhyle as ‘super-synthesis’.

We shall study three cases of super-synthesis using an antenna array. These cases are based on

(a) Observing a radio source towards the celestial North Pole.

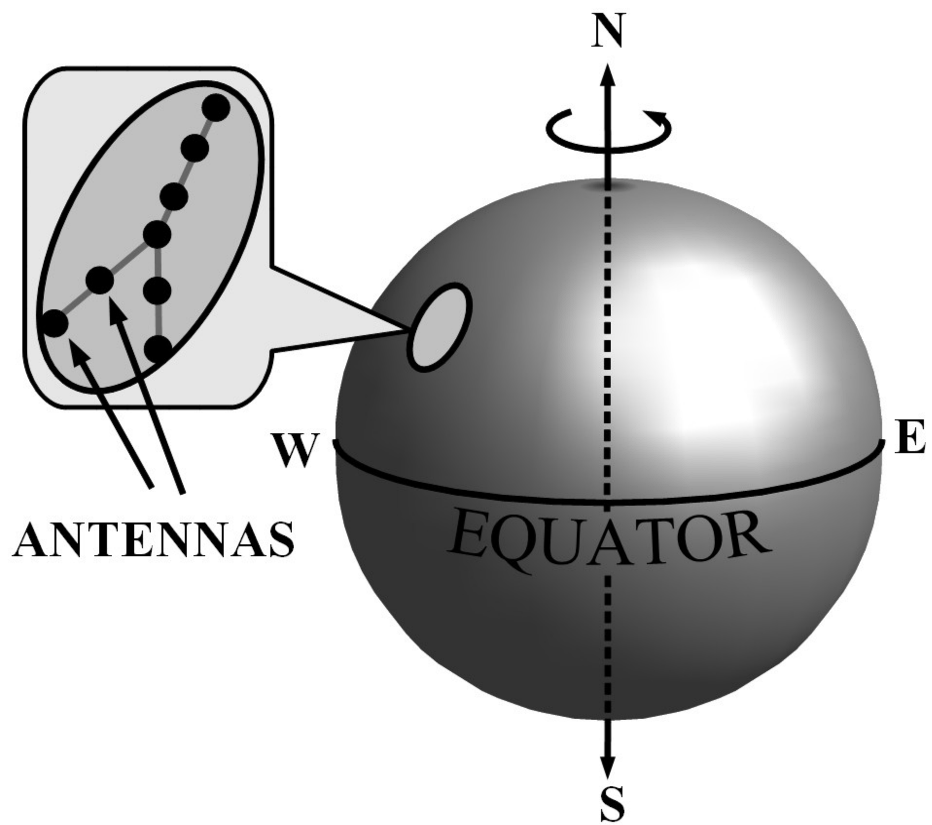
(b) Observing a radio source along the celestial equator.

(c) Observing a radio source along the celestial latitude of the antenna array.

Principles of Aperture Synthesis

Position of the antenna array

Consider a large number of antennas spread over a plane area at some latitude between 20° and 70°. These antennas track a distant radio source located on the celestial North pole. Due to the rotation of Earth, the entire antenna array rotates as seen from the radio source.

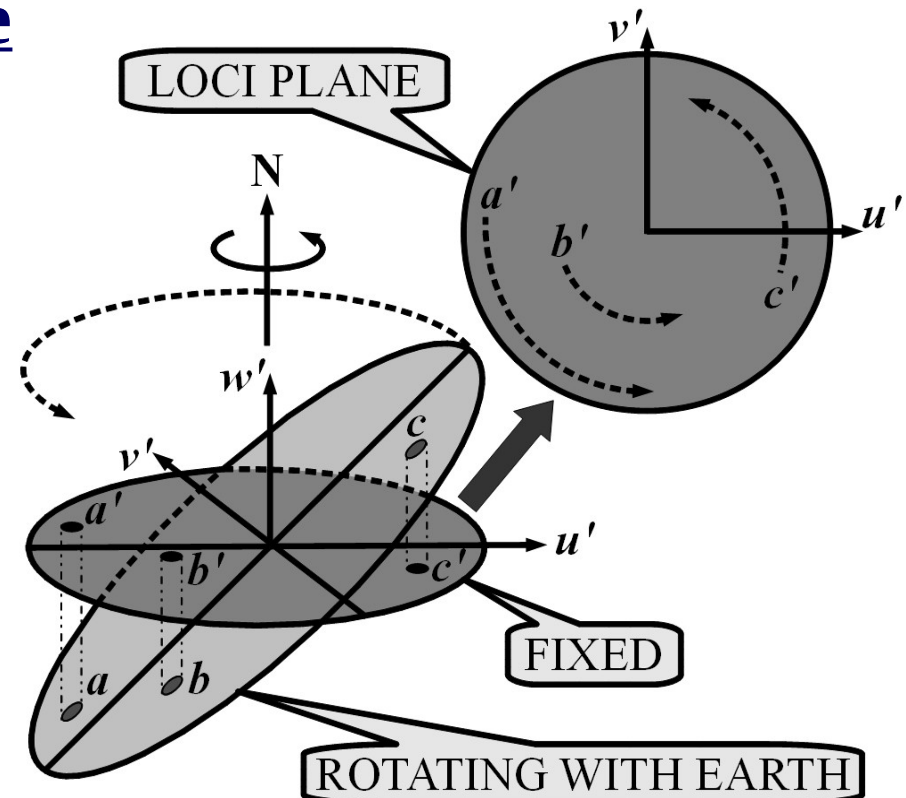


(a)

Principles of Aperture Synthesis

Source at celestial North Pole

Let a rectangular coordinate system (u',v',w') is the origin of center of the plane of antennas. Let the source be at celestial North pole to which w' axis points. Let $u'-v'$ plane be stationary to the source. Due to rotation of Earth, the antennas will appear moving over $u'-v'$ plane as seen from the source.



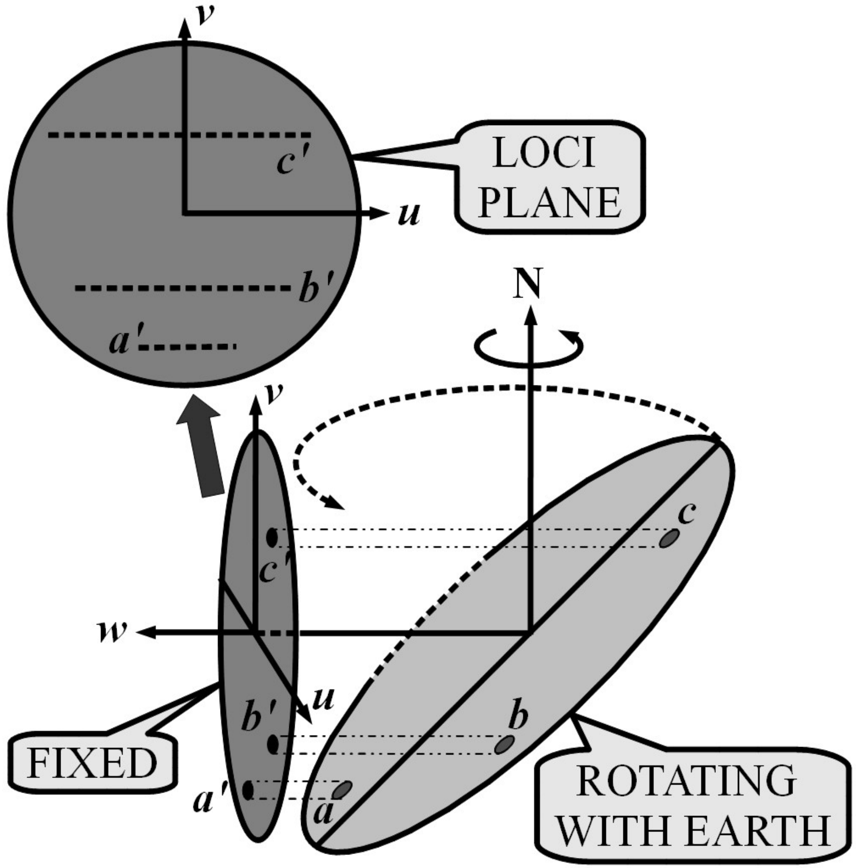
(b)

The loci a', b' and c' of antennas a, b and c on the $u'-v'$ plane will be circles over a period of 24h. We record the output voltage of each antenna and place them on $u'-v'$ plane. The populated area on the $u'-v'$ plane is the effective aperture field.

Principles of Aperture Synthesis

Source at the equator

Here, the loci a', b' and c' of antennas a, b and c on the u−v plane form straight lines. Thus if all the antennas are posited on a single East-West line then all the loci will fall on a single straight line. Hence the population on the u−v plane will be one dimensional, which is not sufficient for making a radio image by Fourier transform.



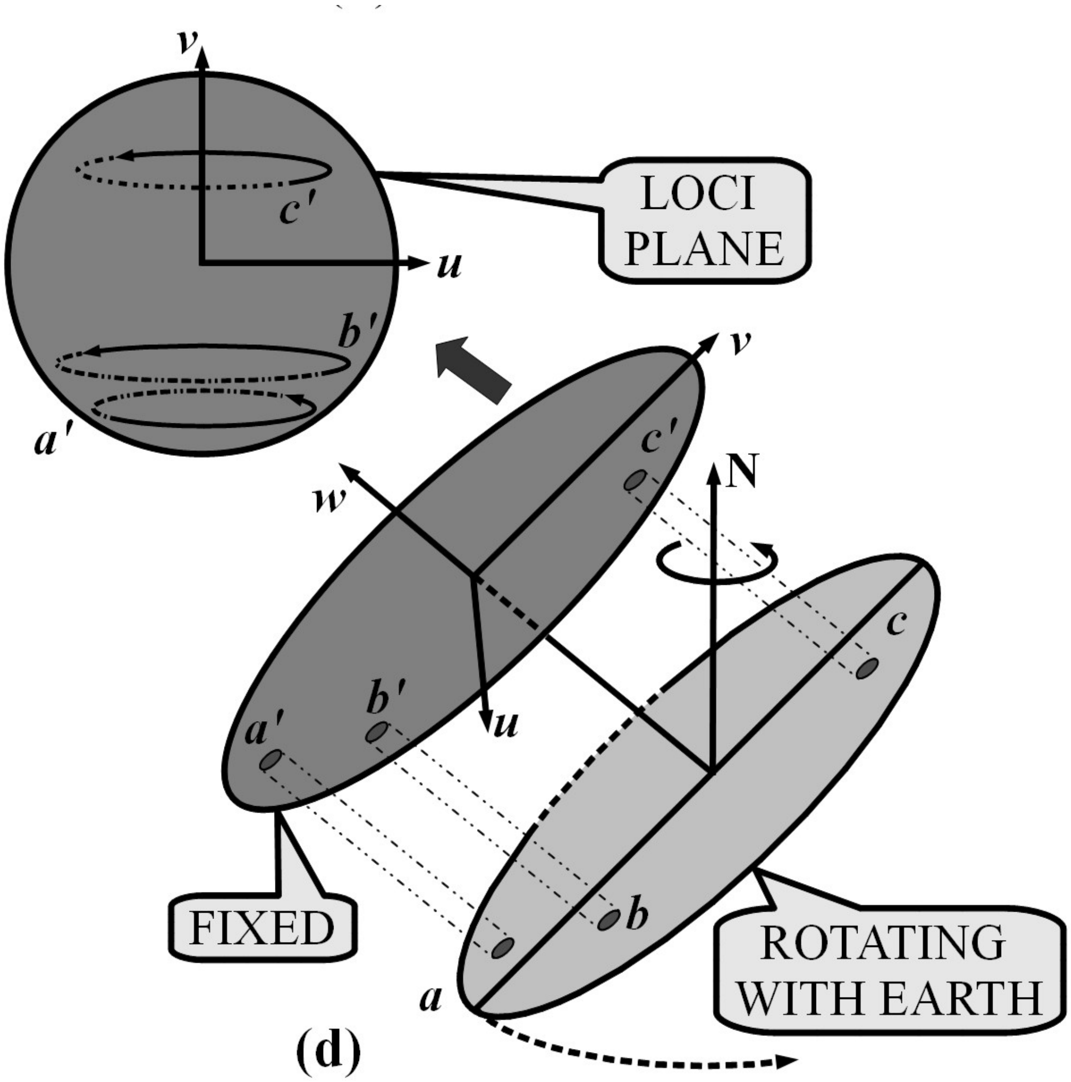
(c)

It is therefore necessary to have some antennas separated along the North-South axis.

Principles of Aperture Synthesis

Source on celestial latitude of antenna array

If the radio source under observation is located at a celestial latitude greater than 0° and less than 90° or within a range greater than -90° and less than 0°, each of the loci will be an ellipse.



(d)

Principles of Aperture Synthesis

For making a radio image we need to do the following operations:

(i) Compute the spatial coherence function known as ***visibility*** from the values of cross correlations $r(\tau_g)$ of the interferometers. The visibility is nothing more than scaled value of cross correlation. It is a function of u,v,w coordinates, represented as $\mathcal{V}\ (u,v,w)$.

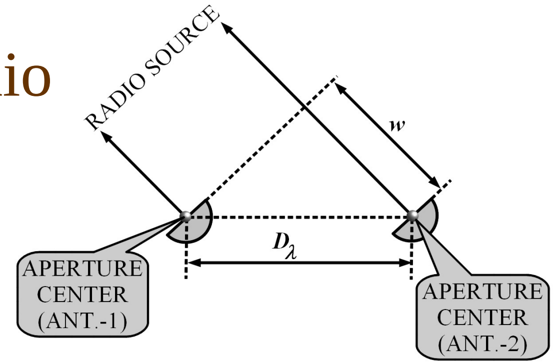
(ii) Place the visibility values on the $u-v$ plane

(iii)Apply a spatial Fourier transformation.

In actual practice there are much more details like calibration and RFI removal. These are performed before the Fourier transformation. The image obtained from Fourier transformation is the dirty image, which are further cleaned with different iterative procedures until the acceptable quality is achieved.

Relating Visibility and Correlation

Consider a simplest case of a two antenna radio interferometer tracking a radio source.



Let the origin of the *l*,*m* coordinates be at phase reference position of the source.

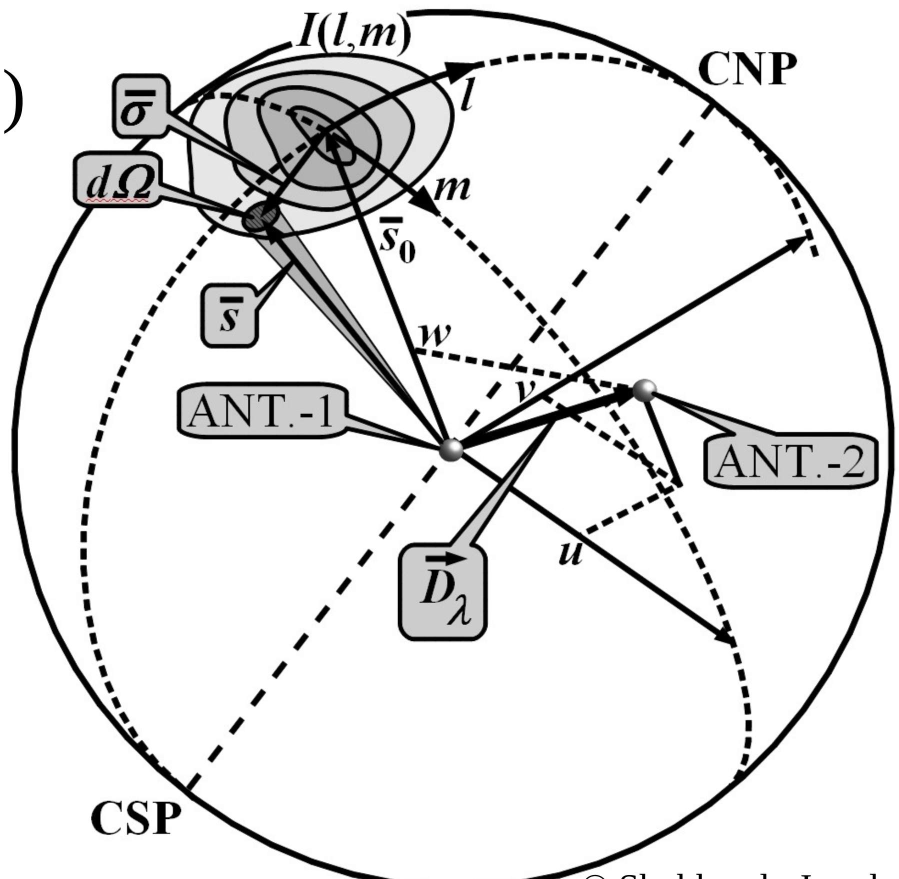
Radio intensity distribution of source on the celestial sphere. $I(l,m)$

Sky intensity/brightness at freq. ν in the direction $\bar{s}$ $I(\bar{s})$

Effective aperture area of an antenna in same direction. $A(\bar{s})$

Then signal power received over a bandwidth $\Delta\nu$ within a solid angular element $d\Omega$ by each antenna is

$$A(\bar{s})\,I(\bar{s})\,\Delta\nu\,d\Omega$$



Relating Visibility and Correlation

Hence, the correlated signal power *dr* per solid angle $d\Omega$ is

$$dr = A(\bar{s})\, I(\bar{s})\, \Delta\nu\, d\Omega \cos(2\pi\nu\tau_g)$$

We integrate *dr* over the celestial sphere and get correlator power *r* as

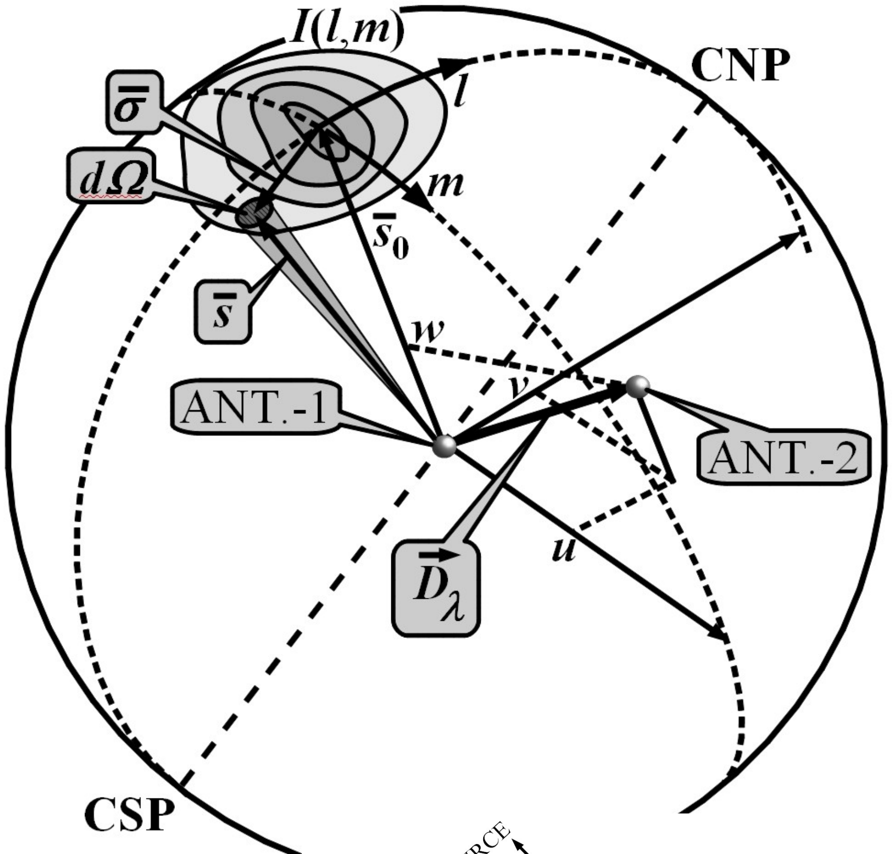
$$r = \Delta\nu \int A(\bar{s})\, I(\bar{s})\, \cos\left[2\pi\left(D_\lambda \cdot \bar{s}\right)\right] d\Omega$$

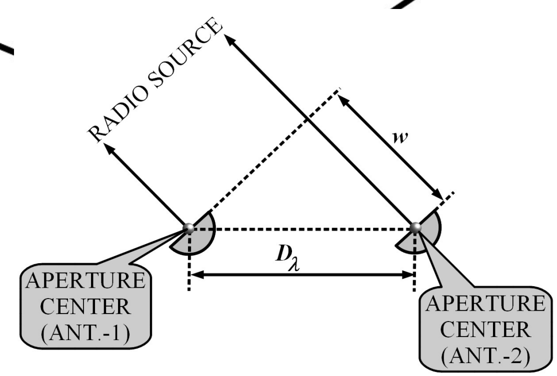
With respect to phase reference position $\bar{s}_0$ we may write $\bar{s}$ as

$$\bar{s} = \bar{\sigma} + \bar{s}_0$$

We may re-express *r* in above terms and expand as

$$r = \Delta\nu \cos\left(2\pi \bar{D}_\lambda \cdot \bar{s}_0\right) \int A\left(\bar{\sigma}\right) I\left(\bar{\sigma}\right) \cos\left(2\pi \bar{D}_\lambda \cdot \bar{\sigma}\right) d\Omega$$
$$- \Delta\nu \sin\left(2\pi \bar{D}_\lambda \cdot \bar{s}_0\right) \int A\left(\bar{\sigma}\right) I\left(\bar{\sigma}\right) \sin\left(2\pi \bar{D}_\lambda \cdot \bar{\sigma}\right) d\Omega$$





Relating Visibility and Correlation

Visibility $\mathcal{V}$ is a complex number

$$V = |V| \exp(j\varphi_v)$$

$$= \int A'(\bar{\sigma})\ I(\bar{\sigma})\ \exp\left(j2\pi \bar{D}_\lambda \cdot \bar{\sigma}\right)\ d\Omega$$

where, A' is normalized aperture pattern (same as normalized beam pattern):

$$A'(\bar{\sigma}) = \frac{A(\bar{\sigma})}{A_0}$$,where, A_0 is max. value.

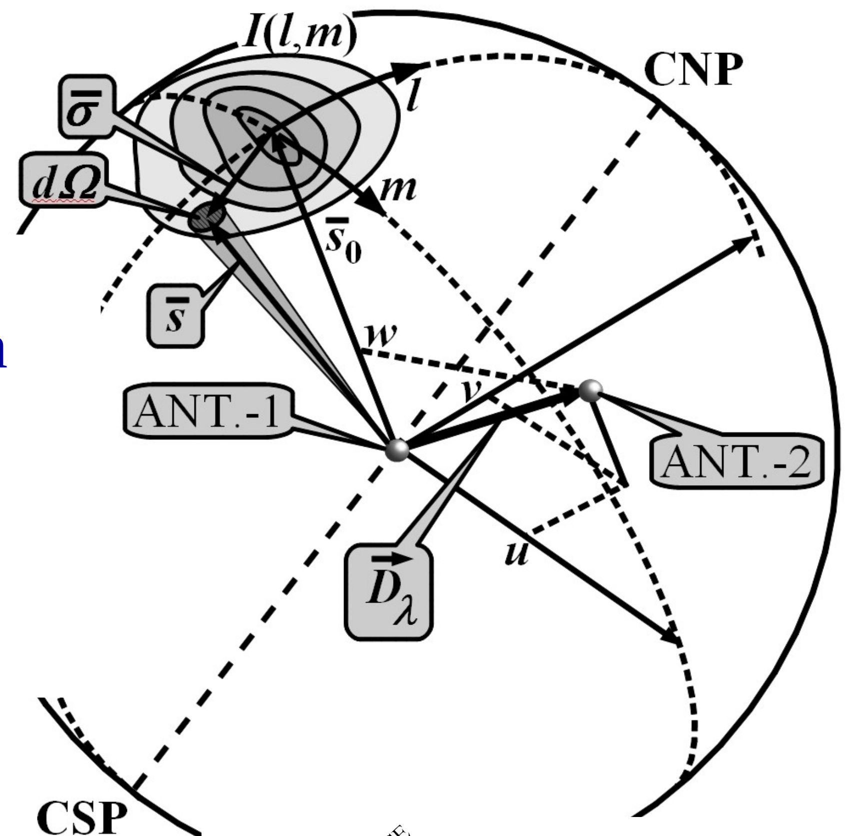
Separating real and imaginary parts

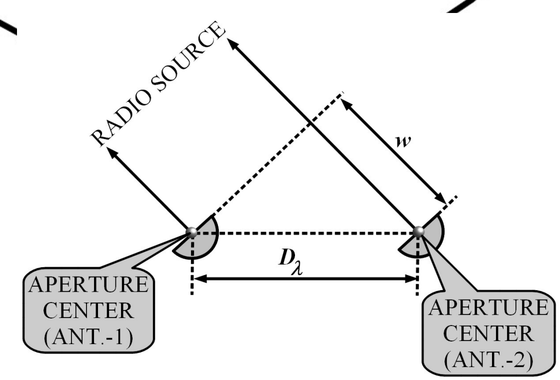
$$|V|\cos\varphi_v = \int A'(\bar{\sigma})\ I(\bar{\sigma}) \cos\left(2\pi \bar{D}_\lambda \cdot \bar{\sigma}\right)\ d\Omega$$

$$|V|\sin\varphi_v = -\int A'(\bar{\sigma})\ I(\bar{\sigma}) \sin\left(2\pi \bar{D}_\lambda \cdot \bar{\sigma}\right)\ d\Omega$$

Thus, the correlation r in terms of visibility is

$$\boxed{r = A_0\ \Delta v\ |V|\ \cos\left(2\ \pi\ \bar{D}_\lambda \cdot \bar{s}_0\right)}$$



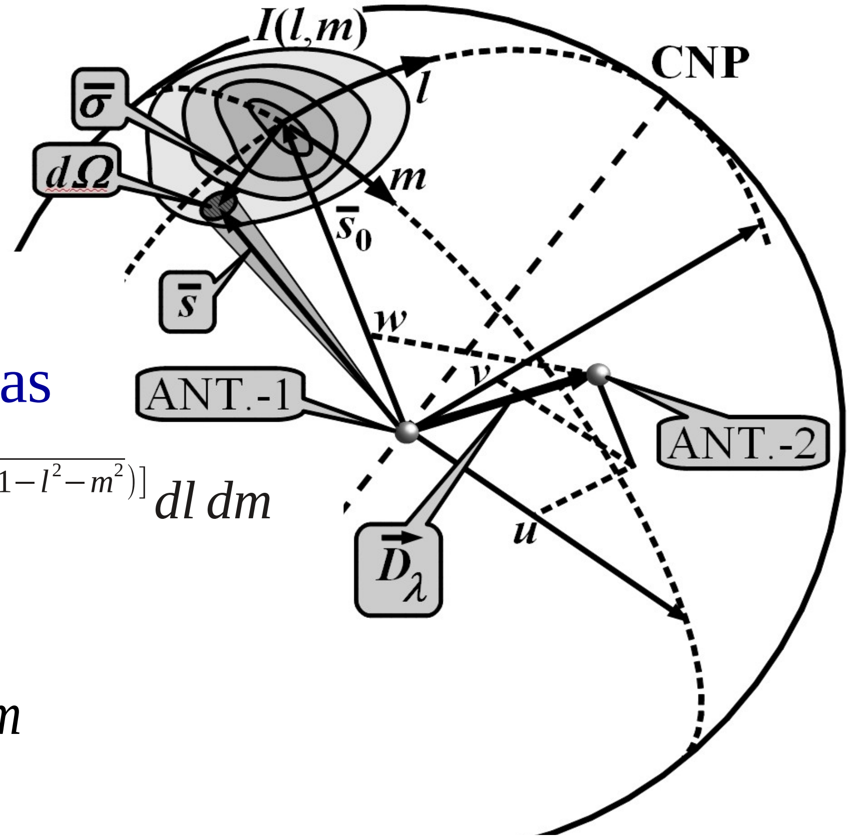


Relating Visibility with Source Intensity

Relation between baseline vector D_λ, observation point, u,v,w and l,m are

$$\bar{D}_\lambda \cdot \bar{s}_0 = w \qquad d\Omega = \frac{dl\ dm}{\sqrt{1-l^2-m^2}}$$

$$\bar{D}_\lambda \cdot \bar{s} = ul + vm + wn$$



Visibility as a function of u,v,w is given as

$$V(u,v,w) = \int\int \frac{A'(l,m)\,I(l,m)}{\sqrt{1-l^2-m^2}}\, e^{-j2\pi[ul+vm+w(\sqrt{1-l^2-m^2})]}\, dl\, dm$$

For a small source $|l|$ and $|m|$ are small

$$\therefore\ w\left(\sqrt{1-l^2-m^2}-1\right) \approx -0.5\left(l^2+m^2\right) << ul+vm$$

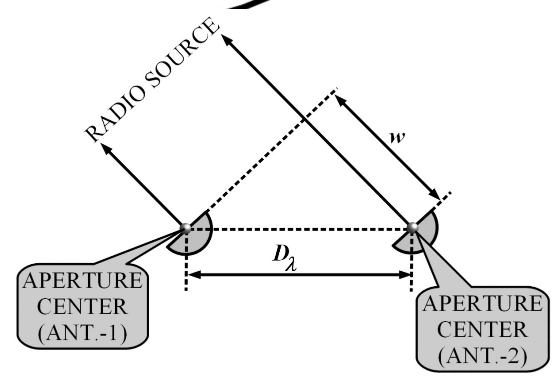
Hence, $V(u,v,w) \approx V(u,v)$

Thus,

$$\boxed{V(u,v) = \iint \frac{A'(l,m)\ I(l,m)}{\sqrt{1-l^2-m^2}}\ e^{-j2\pi(ul+vm)}\, dl\ dm}$$

And its inverse is

$$\boxed{\frac{A'(l,m)\ I(l,m)}{\sqrt{1-l^2-m^2}} = \iint V(u,v)\ e^{j2\pi(ul+vm)}\, du\ dv}$$



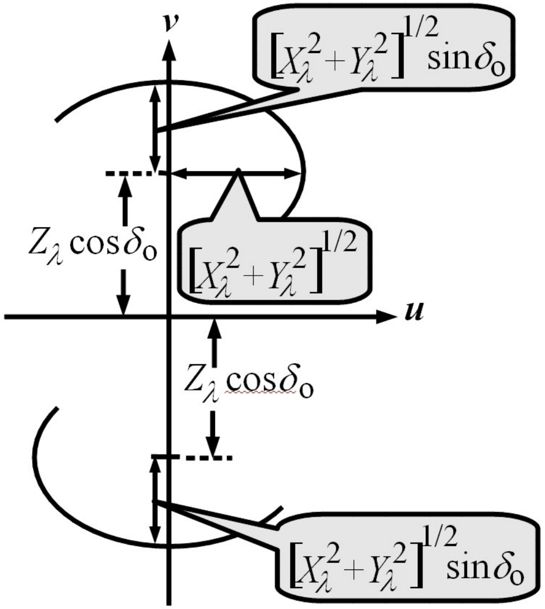
Filling the *u*-*v* plane with visibilities

Recall the transformation of X, Y, Z to u, v, w

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} \sin H & \cos H & 0 \\ -\sin\delta\cos H & \sin\delta\sin H & \cos\delta \\ \cos\delta\cos H & -\cos\delta\sin H & \sin\delta \end{bmatrix} \begin{bmatrix} X_\lambda \\ Y_\lambda \\ Z_\lambda \end{bmatrix}$$

where, $X_\lambda = X/\lambda$, $Y_\lambda = Y/\lambda$, and $Z_\lambda = Z/\lambda$.

Let the phase reference position of radio source under observation be (H_0, δ_0). Thus we have



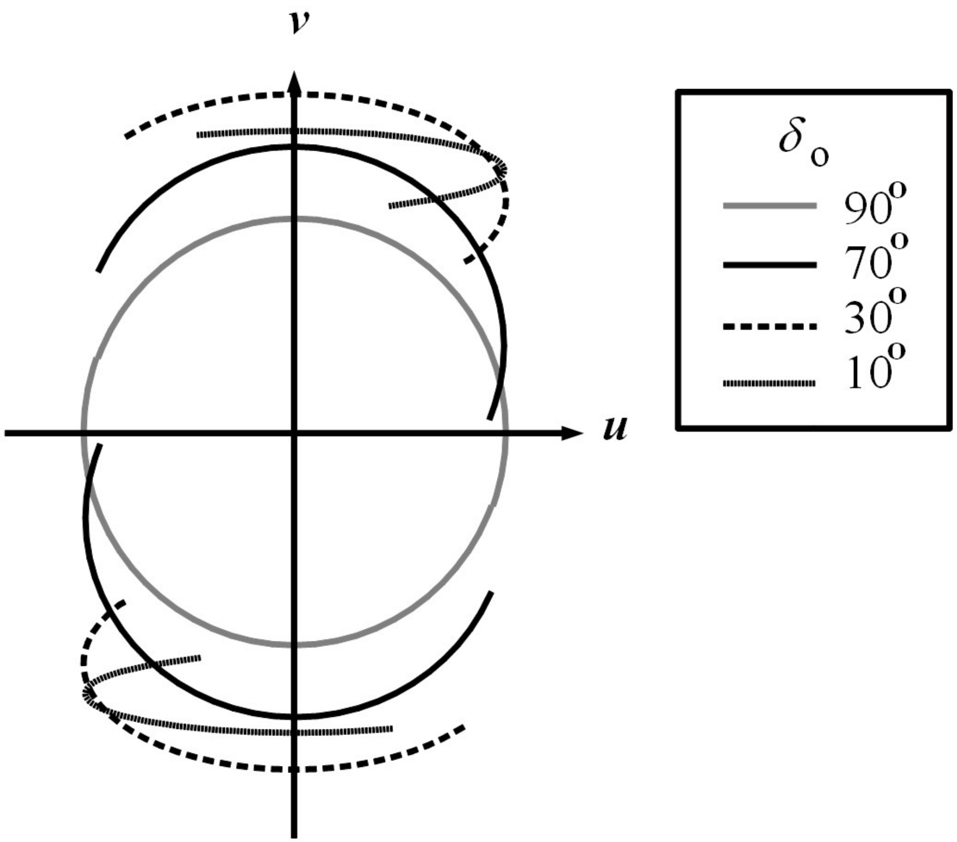
$$u^2 + \left(\frac{v - Z_\lambda\cos\delta_0}{\sin\delta_0}\right)^2 = X_\lambda^2 + Y_\lambda^2$$ **(Ellipse equation)**

This is an ellipse in the u-v plane. However, the ellipse splits into two if Z_λ is not zero, *i.e.*, if a baseline component exists along North-South. Figure shows the locus for a baseline with non-zero Z_λ for a radio source at δ_0. Along u lies the major axis. However, the length of the minor axis along v is reduced by the declination δ_0 of the phase reference position.

Filling the *u*-*v* plane with visibilities

$$u^2 + \frac{v - Z_\lambda \cos\delta_0}{\sin\delta_0} = X_\lambda^2 + Y_\lambda^2$$

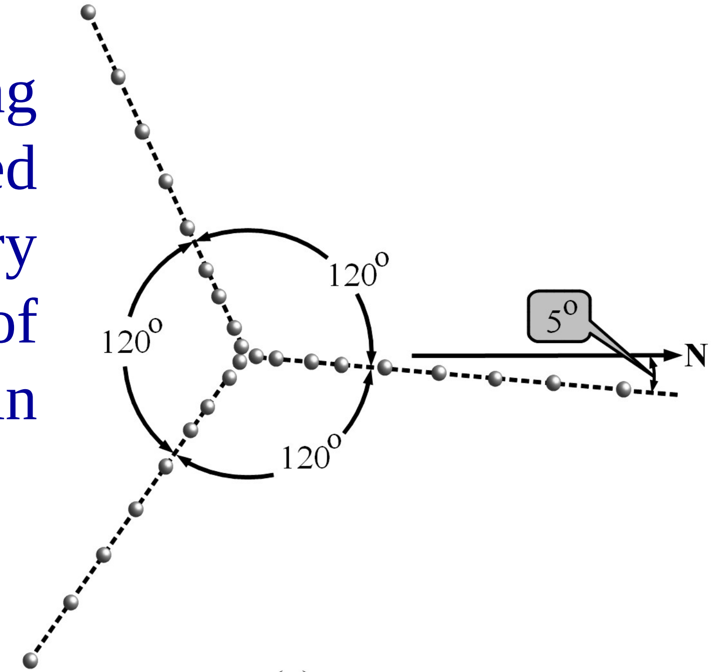
Figure shows the case of a North-South baseline for different phase reference points under observation (for different values of δ_0). If the component of the baseline Z_λ is made zero, the two halves of the ellipse joins together

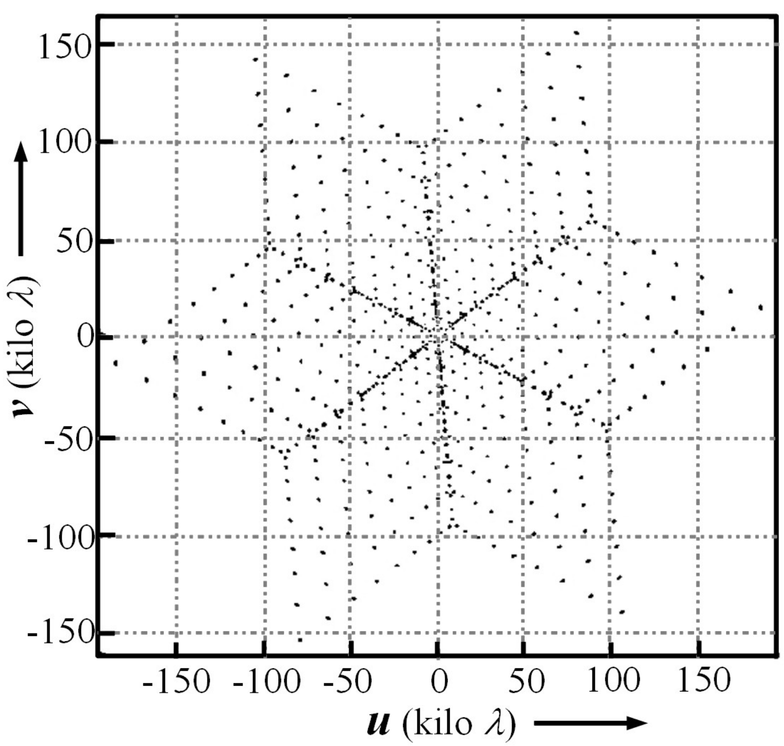


As Earth rotates, depending on δ_0, the locus traces an ellipse on *u*-*v* plane. Visibility values at each instant are placed on the locus. The ellipse axes depend on $D_{\lambda,}$, where, $D_\lambda = (X_\lambda^2 + Y_\lambda^2 + Z_\lambda^2)^{0.5}$. Thus, *u*-*v* plane can be densely filled using (*i*) more interferometers with different D_λ and (*ii*) more observation time.

Filling the *u-v* plane with visibilities

Right figure shows an example of filling the *u*−*v* plane with visibilities obtained from an array of antennas. The VLA (very large array, New Mexico) consists of twenty seven observing antennas posited in Y – configuration.





The number of baselines is $n(n-1)/2$, where n is the number or antennas. If and only if they are non-redundant, the will contribute to *u*-*v* coverage.

Left figure shows the u−v plane coverage by VLA at any instant of time obtained from a source at the celestial North pole.

Making the radio image

The radio image is a map of intensity distribution. After observing for several hours with many antennas, the data on $u-v$ plane becomes adequate. By using

$$\frac{A'(l,m)\ I(l,m)}{\sqrt{1-l^2-m^2}} = \iint V(u,v,w)\, e^{j2\pi\{ul+vm\}} du\, dv$$

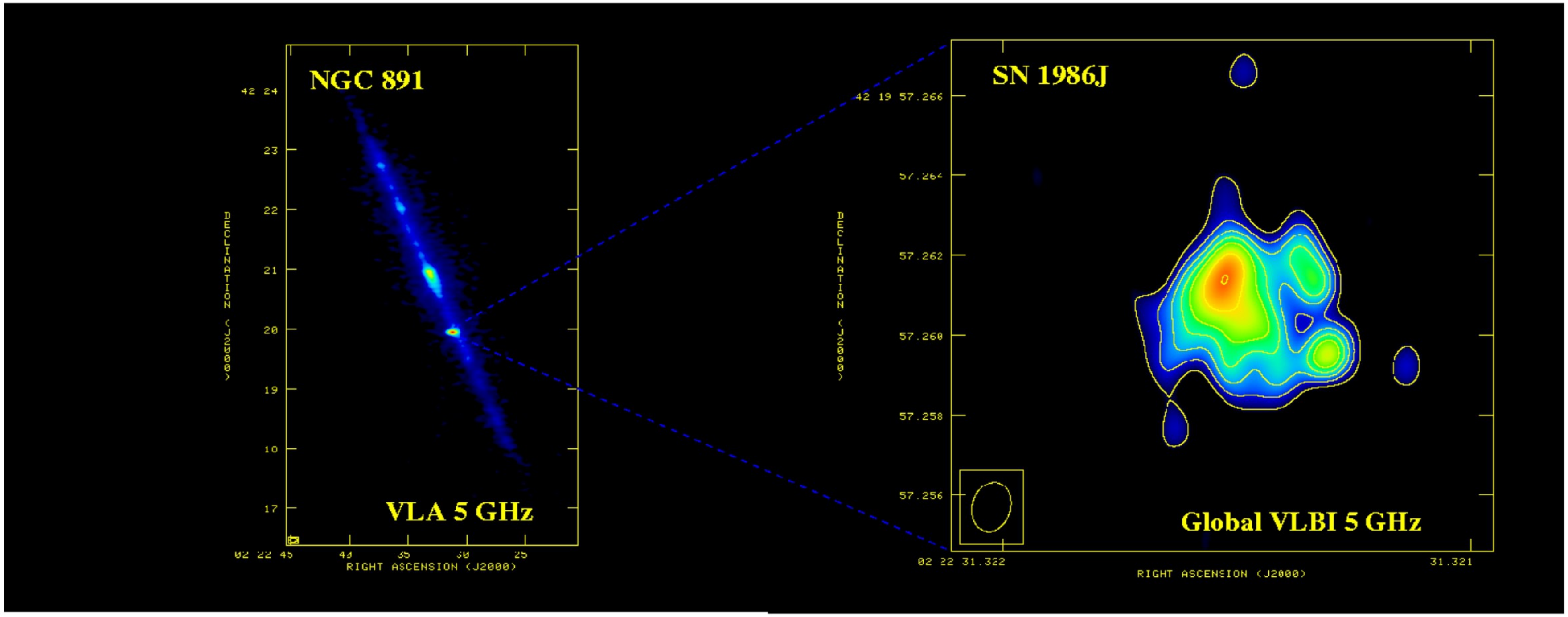
obtain the intensity distribution $I(l,m)$ which is the radio image of the source. However, before such an operation one has to calibrate the data so that it is free from various instrumental errors. Any radio interference and effect of scintillation must be nullified before constructing the image.

Note: Artificial colors are used in radio mapping.

Spiral galaxy M74, in THINGS images.
Credit: Walter et al., NRAO/AUI/NSF.

Correlator Array Radio Images

Data from several distant correlator arrays all over the world may be combined to increase the resolution of the image. This is known as very long baseline interferometry (VLBI).



A supernova SN 1986J within the galaxy NGC 891 taken at 5 GHz. The left is the galaxy NGC 891 using VLA. The brightest red spot is the supernova SN 1986J. The right shows details of the supernova using VLBI. (Copyright NRAO/AUI/NSF).

THANK YOU