Chapter 6

Phased Arrays

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6.1 Introduction

A single element telescope with a steerable paraboloidal reflecting surface is the simplest kind of radio telescope that is commonly used. Such a telescope gives an angular resolution $\sim \lambda/D$, where D is the diameter of the aperture and λ is the wavelength of observation. For example, for a radio telescope of 100 m diameter (which is about the largest that is practically feasible for a mechanically steerable telescope), operating at a wavelength of 1 m, the resolution is $\sim 30\ arc\ min$. This is a rather coarse resolution and is much less than the resolution of ground based optical telescopes.

Use of antenna arrays is one way of increasing the effective resolution and collecting area of a radio telescope. An array usually consists of several discrete antenna elements arranged in a particular configuration. Most often this configuration produces an unfilled aperture antenna, where only part of the overall aperture is filled by the antenna structure. The array elements can range in complexity from simple, fixed dipoles to fully steerable, parabolic reflector antennas. The outputs (voltage signals) from the array elements can be combined in various ways to achieve different results. For example, the outputs may be combined, with appropriate phase shifts, to obtain a single, total power signal from the array – such an array is generally referred to as a phased array. If the outputs are multiplied in distinct pairs in a correlator and processed further to make an image of the sky brightness distribution, the array is generally referred to as a correlator array (or an interferometer). Here we will primarily be concerned with the study of phased arrays, with direct comparison of the performance with correlator arrays, where relevant.

6.2 Array Theory

6.2.1 The 2 Element Array

We begin by deriving the far field radiation pattern for the case of the simplest array, two isotropic point source elements separated by a distance d, as shown in Figure 6.1. The net far field in the direction θ is given as

$$E(\theta) = E_1\, e^{j\psi/2} + E_2\, e^{-j\psi/2} \quad , \tag{6.2.1}$$

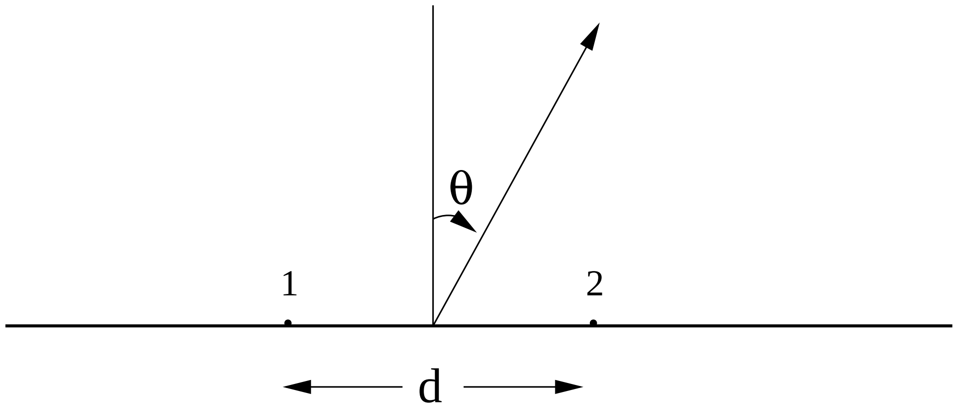

Figure 6.1: Geometry for the 2 element array.

where $\psi = k\,d\,\sin\theta + \delta$, $k = 2\pi/\lambda$ is the wavenumber and δ is the intrinsic phase difference between the two sources. E_1 and E_2 are the amplitudes of the electric field due to the two sources, at the distant point under consideration. The reference point for the phase, referred to as the phase centre, is taken halfway between the two elements. If the two sources have equal strength, $E_1 = E_2 = E_0$ and we get

$$E(\theta) \;=\; 2\,E_0\,\cos(\psi/2) \tag{6.2.2}$$

The power pattern is obtained by squaring the field pattern. By virtue of the reciprocity theorem[1], $E(\theta)$ also represents the voltage reception pattern obtained when the signals from the two antenna elements are added, after introducing the phase shift δ between them.

For the case of $\delta = 0$ and $d \gg \lambda$, the field pattern of this array shows sinusoidal oscillations for small variations of θ around zero, with a period of $2\lambda/d$. Non-zero values of δ simply shift the phase of these oscillations by the appropriate value.

If the individual elements are not isotropic but have identical directional patterns, the result of eqn 6.2.2 is modified by replacing E_0 with the element pattern, $E_i(\theta)$. The final pattern is given by the product of this element pattern with the $\cos(\psi/2)$ term which represents the array pattern. This brings us to the important principle of pattern multiplication which can be stated as : The total field pattern of an array of nonisotropic but similar elements is the product of the individual element pattern and the pattern of an array of isotropic point sources each located at the phase centre of the individual elements and having the same relative amplitude and phase, while the total phase pattern is the sum of the phase patterns of the individual elements and the array of isotropic point sources. This principle is used extensively in deriving the field pattern for complicated array configurations, as well as for designing array configurations to meet specified field pattern requirements (see the book on "Antennas" by J.D. Kraus (1988) for more details).

6.2.2 Linear Arrays of n Elements of Equal Amplitude and Spacing :

We now consider the case of a uniform linear array of n elements of equal amplitude, as shown in Figure 6.2. Taking the first element as the phase reference, the far field pattern is given by

$$E(\theta) \;=\; E_0\left[1 + e^{j\psi} + e^{j2\psi} + \ldots + e^{j(n-1)\psi}\right] \quad , \tag{6.2.3}$$

[1] see Chapter 3

where $\psi = k\,d\sin\theta + \delta$, $k = 2\pi/\lambda$ is the wavenumber and δ is the progressive phase difference between the sources. The sum of this geometric series is easily found to be

$$E(\theta) = E_0 \frac{\sin(n\psi/2)}{\sin(\psi/2)}\, e^{j(n-1)\psi/2}. \tag{6.2.4}$$

If the centre of the array is chosen as the phase reference point, then the above result does not contain the phase term of $(n-1)\psi/2$. For nonisotropic but similar elements, E_0 is replaced by the element pattern, $E_i(\theta)$, to obtain the total field pattern.

The field pattern in eqn 6.2.4 has a maximum value of nE_0 when $\psi = 0, 2\pi, 4\pi, \ldots$. The maxima at $\psi = 0$ is called the main lobe, while the other maxima are called grating lobes. For $d < \lambda$, only the main lobe maxima maps to the physically allowed range of $0 \le \theta \le 2\pi$. By suitable choice of the value of δ, this maxima can be "steered" to different values of θ, using the relation $k\,d\sin\theta = -\delta$. For example, when all the elements of the array are in phase ($\delta = 0$), the maximum occurs at $\theta = 0$. This is referred to as a "broadside" array. For a maximum along the axis of the array ($\theta = 90^o$), $\delta = -k\,d$ is required, giving rise to an "end-fire" array. The broadside array produces a disc or fan shaped beam that covers a full 360^o in the plane normal to the axis of the array. The end-fire array produces a cigar shaped beam which has the same shape in all planes containing the axis of the array. For nonisotropic elements, the element pattern also needs to be steered (electrically or mechanically) to match the direction of its peak response with that of the peak of the array pattern, in order to achieve the maximum peak of the total pattern.

For the case of $d > \lambda$, the grating lobes are uniformly spaced in $\sin\theta$ with an interval between adjacent lobe maxima of λ/d, which translates to $\ge \lambda/d$ on the θ axis (see Figure 6.3).

The uniform, linear array has nulls in the radiation pattern which are given by the condition $\psi = \pm 2\pi l/n,\ \ l = 1, 2, 3, \ldots$ which yields

$$\theta = \sin^{-1}\left[\frac{1}{kd}\left(\pm\frac{2\pi l}{n} - \delta\right)\right] . \tag{6.2.5}$$

For a broadside array ($\delta = 0$), these null angles are given by

$$\theta = \sin^{-1}\left(\pm\frac{2\pi l}{nkd}\right) . \tag{6.2.6}$$

Further, if the array is long ($nd \gg l\lambda$), we get

$$\theta \simeq \pm\frac{\lambda l}{nd} \simeq \pm\frac{l}{L_\lambda} , \tag{6.2.7}$$

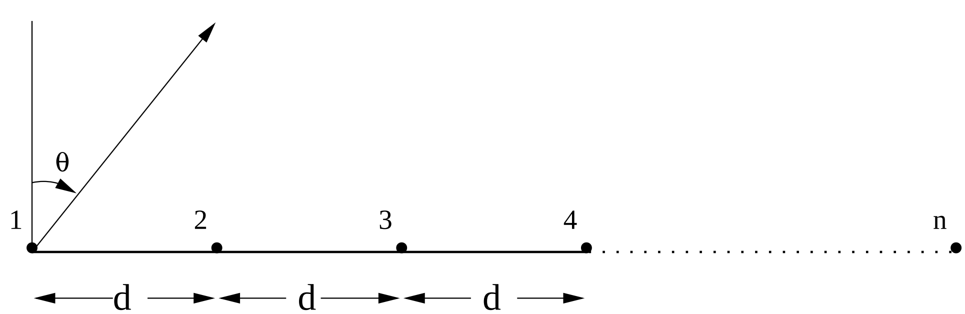

Figure 6.2: Geometry for the n element array

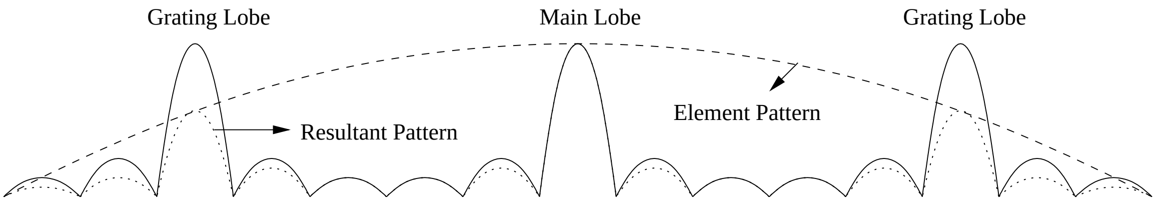

Figure 6.3: Grating lobes for an array of n identical elements. The solid line is the array pattern. The broad, dashed line curve is an example of the element pattern. The resultant of these two is shown as the dotted pattern.

where L_λ is the length of the array in wavelengths and $L_\lambda = (n-1)d/\lambda \simeq nd/\lambda$ for large n. The first nulls occur at $l = \pm 1$, and the beam width between first nulls (BWFN) for such an array is given by

$$BWFN \;=\; \frac{2}{L_\lambda}\, rad \;=\; \frac{114.6}{L_\lambda}\, deg \;. \tag{6.2.8}$$

The half-power beam width (HPBW) is then given by

$$HPBW \;\simeq\; \frac{BWFN}{2} \;=\; \frac{57.3}{L_\lambda}\, deg \;. \tag{6.2.9}$$

Similarly, it can be shown that the HPBW of an end-fire array is $\sqrt{2/L_\lambda}$ (see "Antennas" by J.D. Kraus (1988) for more details).

Such linear arrays are useful for studying sources of size $< \lambda/d$ radians, as only one lobe of the pattern can respond to the source at a given time. Also, the source should be strong enough so that confusion due to other sources in the grating lobes is not significant. Linear grating arrays are particularly useful for studying strong isolated sources such as the Sun.

The presence of grating lobes (with amplitude equal to the main lobe) in the response of an array is usually an unwanted feature, and it is desirable to reduce their levels as much as possible. For non-isotropic elements, the taper in the element pattern provides a natural reduction of the amplitude of the higher grating lobes. This is illustrated in Figure 6.3. To get complete cancellation of all the grating lobes starting with the first one, requires an element pattern that has periodic nulls spaced λ/d apart, with the first null falling at the location of the first grating lobe. This requires the elements to have an aperture of $\sim d$, which makes the array equivalent to a continuous or filled aperture telescope. This can be seen mathematically by replacing E_0 in eqn 6.2.4 by the element pattern of an antenna of aperture size d and showing that it reduces to the expression for the field pattern of a continuous aperture of size nd.

The theoretical treatment given above is easily extended to two dimensional antenna arrays.

6.2.3 The Fourier Transform Approach to Array Patterns

So far we have obtained the field pattern of an array by directly adding the electric field contributions from different elements. Now, it is well established that for a given aperture, if the electric field distribution across the aperture is known, then the radiation pattern can be obtained from a Fourier Transform of this distribution (see, for example, Christiansen & Hogbom 1985). This principle can also be used for computing the field pattern of an array. Consider the case of the array pattern for the 2-element array discussed earlier, as an example. The electric field distribution across the aperture can be taken to be zero at all points except at the location of the two elements, where it is a delta function for isotropic point sources. The Fourier Transform of this gives the sinusoidal oscillations in $\sin\theta$, which have also been inferred from eqn 6.2.2.

Using the Fourier Transform makes it easy to understand the principle of pattern multiplication described above. When the isotropic array elements are replaced with directional elements, it corresponds to convolving their delta function electric field distribution with the electric field distribution across the finite apertures of these directional elements. Since convolution of two functions maps to multiplication of their Fourier Transforms in the transform domain, the total field pattern of the array is naturally the product of the field pattern of the array with isotropic elements with the field pattern of a single element. The computational advantages of the Fourier Transform makes this approach the natural way to obtain the array pattern of two dimensional array telescopes having a complicated distribution of elements.

6.3 Techniques for Phasing an Array

The basic requirement for phasing an array is to combine the signals from the elements with proper delay and phase adjustments so that the beam can be pointed or steered in the chosen direction. Some of the earliest methods employed techniques for mechanically switching in different lengths of cables between each element and the summing point, to introduce the delays required to phase the array for different directions. The job became somewhat less cumbersome with the use of electronic switches, such as PIN diodes. However, the complexity of the cabling and switching network increases enormously with the increase in number of elements and the number of directions for which phasing is required.

Another method of phasing involves the use of phase shifters at each element of the array. For example, this can be achieved by using ferrite devices or by switching in incremental lengths of cable (or microstrip delay lines), using electronic switches. The phase increments are usually implemented in binary steps (for example $\lambda/2$, $\lambda/4$, $\lambda/8, \ldots$). In this scheme, the value of the smallest incremental phase difference controls the accuracy of the phasing that can be achieved.

In most modern radio telescopes, digital electronic techniques are used for processing the signals. The output from an antenna is usually down-converted to a baseband frequency in a heterodyne receiver after which it is Nyquist sampled for further processing. Techniques for introducing delays and phase changes in the signal in the digital domain, using computers or special purpose hardware, are fairly easy to implement and flexible.

The description of phasing techniques given above applies when the delay compensation of the signals from the different elements of the array is carried out at the radio frequency of observation. When this delay compensation is carried out at the intermediate or baseband frequency of the heterodyne receiver, the signals pick up an extra phase term of $2\pi\,\nu_{LO}\,\tau_g$, where ν_{LO} is the local oscillator frequency used for the down conversion and τ_g is the delay (with respect to the phase centre of the array) suffered for the element (see for example Thompson, Moran & Swenson, 1986). To obtain the optimum phased array signal, these phase terms have to be compensated before the signals from array elements with different values of τ_g are added. Furthermore, τ_g for an array element varies with time for observations of a given source and this also needs to be compensated.

For an array with similar elements, the amplitude of the signals from the elements is usually kept constant at a common value, while the phase is varied to phase the array. However, in the most general case, the amplitude of the signals from different elements can be adjusted to enhance some features of the array response. This is most often used to reduce the sidelobe levels of the telescope or shift the nulls of the array pattern to desired locations, such as directions from which unwanted interference signals may be coming. Arrays where such adjustments are easily and dynamically possible are called adaptive beam-forming arrays, and are discussed further in Chapter 7.

6.4 Coherently *vs* Incoherently Phased Array

Normally, the signals from an n-element phased array are combined by adding the voltage signals from the different antennas after proper delay and phase compensation. This summed voltage signal is then put through a square-law detector and an output proportional to the power in the summed signal is obtained. For identical elements, this phased array gives a sensitivity which is n times the sensitivity of a single element, for point source observations. The beam of such a phased array is much narrower than that of the individual elements, as it is the process of adding the voltage signals with different phases from the different elements that produces the narrow beam of the array pattern. For some special applications, it is useful to first put the voltage signal from each element of the array through a square-law detector and then add the powers from the elements to get the final output of the array. This corresponds to an incoherent addition of the signals from the array elements, whereas the first method gives a coherent addition. In the incoherent phased array operation, the beam of the resultant telescope has the same shape as that of a single element, since the phases of the voltages from individual elements are lost in the detection process. This beam width is usually much more than the beam width of the coherent phased array telescope. The sensitivity to a point source is higher for the coherent phased array telescope as compared to the incoherent phased array telescope, by a factor of $\sqrt{n}$.

The incoherent phased array mode of operation is useful for two kinds of astronomical obervations. The first is when the source is extended in size and covers a large fraction of the beam of the element pattern. In this case, the incoherent phased array observation gives a better sensitivity. The second case is when a large region of the sky has to be covered in a survey mode (for example, in a survey of the sky in search for new pulsars). Here, the time taken to cover the same area of sky to equal senstivity level is less for the incoherent phased array mode. Only for a filled aperture phased array telescope are these times the same. For a sparsely filled physical aperture such as an earth rotation aperture synthesis telescope, this distinction between the coherent and incoheret phased array modes is an important aspect of phased array operation.

6.5 Comparison of Phased Array with a Multi-Element Interferometer

As has been mentioned in Section 1, the basic distinction between a phased array and a multi-element interferometer is that in a phased array the signals from all the elements are added in phase before (or after) being put through a square-law detector, where as in a multi-element interferometer, the signals from the elements are correlated in pairs for each possible combination of two elements and these outputs are further processed to make a map of the brightness distribution. Thus, if the signal from element *i* is given by V_i, the output of the (coherent) phased array can be written as

$$V_{PA} = \left\langle \left(\sum_{i=1}^{n} V_i \right)^2 \right\rangle \quad (6.5.10)$$

whereas the interferometer output is given by

$$V_{ij} \quad = \quad \langle V_i \, V_j \rangle \quad i,j = 1,2,\ldots,n\,;\, i \neq j \tag{6.5.11}$$

Expansion of the right hand side of eqn 6.5.10 produces terms of the kind $< V_i \, V_j >$ and V_i^2. The first kind are all available from the correlator outputs and, if the correlator also records the self products of all the elements, the second kind are also provided by the correlator. Thus, by appropriate combinations of the outputs of the correlator used in the multi-element interferometer, the phased array output can be synthesised. Even the steering of the beam of the phased array can be achieved by combining the visibilities from the correlator after multiplying with appropriate phase factors. Also, the incoherently phased array output can be synthesised by combining only the self product outputs from the correlator.

However, the network of multipliers required to implement the correlator is a much more complicated hardware than the adder and square law detector needed for the phased array. Further, the net data rate out of the correlator is much higher than that from the phased array output, for data with the same time resolution. Thus, the interferometer achieves the phased array response in a very expensive manner. This is especially true for very compact, point-like sources where observations with an interferometer do not provide any extra information about the nature of the source. For example, observations of pulsars are best suited to a phased array, as these are virtually point sources for the interferometer and the requirement for high time resolution that is relevant for their studies is more easily met with a phased array output.

6.6 Further Reading

1. Kraus, J.D. "Radio Astronomy", Cygnus-Quasar Books, Ohio, USA, 1986
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3. Thompson, A.R., Moran, J.M. & Swenson, G.W. "Interferometry and Synthesis in Radio Astronomy", John Wiley & Sons, New York, USA, 1986
4. Christiansen, W.N. & Hogbom, J.A. "Radio Telescopes", Cambridge University Press, Cambridge, UK, 1985